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Abstract:
We present a model of endogenous firm growth with R&D investment and innovation as the engine of growth. The objective of our analysis is to present a framework that can be used for microeconometric analysis of firm performance in high-tech industries. The model for firm growth is a partial equilibrium model drawing on the quality ladder models in the macro growth literature, but also on the literature on patent races and the discrete choice models of product differentiation. We examine to what extent the assumptions and the empirical content of our model are consistent with the findings that have emerged from empirical studies of growth, productivity, R&D and patenting at the firm level. The analysis shows that the model fits well empirical patterns such as (i) a skewed size distribution of firms with persistent differences in firm sizes, (ii) firm growth (roughly) independent of firm size (the so-called Gibrat's law) and (iii) R&D investment proportional to sales, as well as a number of other empirical patterns.

Keywords: Firm growth, R&D-investment, Gibrat's law, Product innovations

JEL classification: L11, O32, D92

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1 Few theories on firm growth and firm heterogeneity

Empirical research on firm and plant level data has revealed a large amount of heterogeneity within narrowly defined industries. This heterogeneity is striking in a number of dimensions such as size (sales, employment), firm growth rates, rates of job creation and job destruction, and also in variables such as capital intensity and R&D intensity[1]. Much of the heterogeneity, e.g. in terms of sales and R&D investment, is quite persistent over a number of years. Recently, researchers have addressed the question of how we can reconcile this persistent heterogeneity with theories based on optimizing agents (see survey by Sutton, 1996). In this paper, we present a model of endogenous firm growth where R&D investment and stochastic innovation are the engines of growth. Throughout our analysis, we examine to what extent our model is consistent with the empirical evidence on firm growth and the findings from the microeconometric research on R&D, innovation and patents.

Our model can be made consistent, for specific parameter values, with at least three widely studied empirical regularities of R&D investment, firm sales and firm growth: (i) R&D intensities are (roughly) independent of sales[2]. This result is derived from our model which treats R&D investment as a *non-rival* input in production. (ii) Firm growth is (to a first approximation) independent of size. This relationship is often referred to as Gibrat's law[3]. (iii) The size distribution of firms is highly skewed with *persistent differences* in firm sizes. This is true both for sales and other variables such as R&D investment. This third proposition is closely related to proposition (ii), as has been emphasized by Simon and his co-workers (see Ijiri and Simon, 1977). Our contribution is to show that these three propositions can all be related and derived from a fully specified model of endogenous firm growth, based on optimizing agents. We examine whether the empirical content of the assumptions required to obtain these propositions make sense in view of what we have learned from microeconometric research on innovation, patents and R&D. We also consider a number of additional empirical implications that we derive from our model.

Our objective is to develop a model that can serve as a framework for firm level analysis of R&D investment, innovation, patenting and firm performance in terms of profitability and growth. In the present paper, we discuss the empirical implications of our model in relation to results from microeconometric studies presented elsewhere. In a companion (*draft*) paper, Klette

[1]Some recent contributions to this empirical research are e.g. Dunne et al. (1988), Mairesse and Griliches (1990), and Davis and Haltiwanger (1992). The large literature on the skewed distribution of firm sizes is surveyed in Sutton (1996).

[2]See Cohen and Klepper (1996) for a recent review of this issue.

[3]See Sutton (1996) for a comprehensive survey.

and Griliches (1996), we consider more closely the empirical performance of our model, using line-of-business level (within firms) data for firms in high-tech industries[4]. The companion paper examines the performance of the model in a number of dimensions simultaneously, allowing us to throw light on omitted parts of the model that will show up in the cross correlation of the residuals in the different relationships (estimating equations).

Our model is an "extreme hypothesis" in Simon's (1968) terminology; we do not, of course, expect the model to fit all relevant empirical facts[5]. We consider our model a *benchmark* for empirical analysis, and we discuss some extensions of the framework in the final section.

The specification of our model is inspired by the macro-models of endogenous growth, in particular the quality ladder model developed by Grossman and Helpman (1991a,b) and Aghion and Howitt (1992). Their version of the quality ladder model implies that each new innovation is introduced by a new firm. Barro and Sala-i-Martin (1995, ch.7) and Thompson (1996) point out that it is hard to reconcile this property with the observed pattern with persistent dominance of established firms (at least on an annual time scale). We have briefly discussed the previous literature on the quality ladder model in appendix A, where we also list a number of other studies related to our analysis. Ericson and Pakes (1995) study has a focus similar to ours in that they develop a theoretical model of R&D investment and firm growth through innovation consistent with a number of empirical facts. Another related study is Cohen and Klepper (1996) which provides an interpretation of the empirical patterns of R&D investment similar to our model. See appendix A for further remarks on these and other studies.

The next section spells out the model and discusses the validity of the assumptions of the model in view of existing empirical studies. Additional empirical implications of the model are derived and examined in section 3. Section 4 elaborates on the specification of the demand side of our model, the nature of price competition and the optimal price setting, which are treated rather briefly in section 2. Section 5 provides conclusions and discussion of future research tasks.

2 Our version of the quality ladder model

2.1 The quality ladder model, macro growth and firm growth

This section spells out the basic structure of the model, and we discuss the empirical content of the assumptions in view of available results from microeconometric studies of firm performance,

[4]Line-of-business level data are better for testing our framework than firm level data, since many of the R&D intensive firms are conglomerates of activities in many different product fields (both low and high tech), and the growth mechanism emphasized in this paper is likely to be too crude to study growth of such conglomerate firms.

[5]This issue reminds us of a quotation from Picasso, who is supposed to have said that: "Art is the lie that help us see the truth". We accept that our model - as any model - is "a lie", but we also hope that it can help us to reveal true insights about firm growth and performance.

R&D and innovation. As pointed out above, our analysis is a variation of the quality ladder model, with elements from the patent race literature concerned with the persistence of monopoly. We have replaced the specification of the demand side in the quality ladder models of Grossman-Helpman (1991a, 1991b) and Barro-Sala-i-Martin (1995, ch.7), with a specification based on the discrete choice theory for differentiated products, as presented e.g. by Anderson, dePalma and Thisse (1992) and Berry (1994). Our alternative specification of the demand side allows the model to account for both the *growth and decline* of firms. Section 4 discusses the properties of this demand system in more detail.

2.2 Alternative specification of the demand side

Consider a firm producing a product "*i*" of quality k_i and with price P_i.

Assumption 1.A: *The firm's sales are determined by*

$$S_i = M \frac{e^{v(k_i, P_i, \omega_i)}}{\sum_{j \in J} e^{v(k_j, P_j, \omega_j)}} \tag{1}$$

with $v(k_i, P_i, \omega_i) = \gamma k_i - \alpha \ln P_i + \omega_i$. γ and α are parameters and M *is the size of the market, while* ω_i *captures unobservable demand shifters, apart from quality.* J *refers to all the competitors in the market (including foreign competitors).*

This demand system has been extensively discussed by others; see e.g. Anderson, dePalma and Thisse (1992) and Berry (1994) and references cited in those papers.

Assumption 1.B: *Firm profits are determined according to the formula*

$$\Pi_i = \pi_i S_i$$

where π_i *is a fixed profit margin per unit of sales, while* S_i *is sales as determined in (1).*

That π_i is fixed, or determined independently of sales (S_i), is a restrictive assumption which simplifies the analysis of (optimal) R&D investment considerably. We discuss this assumption and how it can be relaxed in section 4.

In the following, we will refer to the sales function $S_i = S(k_i, \Psi)$, i.e. as a function of the product quality k_i and other variables. These other variables are represented by Ψ, capturing the impact of the firm's product price and the prices and qualities of the competing products (cf. equation 1). The firm considers Ψ as determined independently of k_i. Below, where we model R&D investment and firm growth, we will focus on k_i as a determinant of demand; it is changes in k_i that are affected by R&D investment and that largely determines firm growth. We will elaborate on the empirical content of assumptions 1.A and 1.B in section 4.

2.3 Dynamic optimization and firm growth

Let us now focus on one particular differentiated product, and the process of quality upgrading through R&D of this product. Each step in this upgrading involves a three stage game. In the first stage, the incumbent determines R&D investment in its product line. In stage two, there is free entry by outside competitors into the product line. When the new upgraded version of the product is developed, the game enters the third stage, with price competition between all the differentiated product lines (firms) in the industry, conditional on the prevailing product qualities. In this setup, we should emphasize the assumption:

Assumption 2: *The incumbent decides its level of R&D investment before potential entrants.*

As clarified in the exchange between Gilbert-Newbery and Reinganum in the *American Economic Review*, this assumption about the order of moves has empirical content; see Gilbert and Newbery (1984). It is required to favor the persistent dominance of the incumbent. We pointed out above, that at least at an annual scale, there is a large empirical literature that has documented the persistency of leading firms (in terms of size); cf. Sutton's survey (Sutton, 1996)[6]. The process of "creative destruction", where one dominant firm is replaced by another, is emphasized both by Reinganum (1985) in her model of industry evolution, and in the quality ladder models by Grossman-Helpman (1991a, 1991b) and Aghion-Howitt (1992). Such a process of creative destruction might be more relevant for the longer run (say, in terms of decades rather than years). Hall, Griliches and Hausman (1986) has documented the persistency of differences in R&D effort between firms in the same industry[7]. We will return to this question below.

We will now present a formal analysis of optimal R&D investment that leads us to a model of firm growth. Consider one particular firm producing a given product with a certain quality level. In our model, quality improvements for this product will take place through R&D effort. The firm with its product of quality k earns profit $\Pi(k)$ per unit of time. We will suppress the firm subscript and the other variables entering the profit function to avoid notational clutter; these other variables include prices and quality variables for the competitors. Assume that the product is replaced by a new product after a period $T(k)$, where $T(k)$ is a stochastic variable. It follows that the value of getting the new product, conditional on $T(k)$, is given by

$$V(k;T) = \Pi(k)\ (1 - e^{-rT(k)})/r \tag{2}$$

[6]See also the discussion in Gruber (1992) and Klepper (1996).

[7]One should notice that the persistency of R&D effort documented in Hall, Griliches and Hausman (1986) is somewhat exaggerated, since their analysis is based on the continuing and hence heavily selected panel.

The arrival time of the next product $(k+1)$ is uncertain. Denote the probability of the arrival of the next product per unit of time by $\lambda(k)$, which is some function of R&D investment and the quality of the existing product, i.e. $\lambda(k) = h(R,k)$. We expect $h_1 > 0$, while we will argue below that h_2 is negative. The cumulative distribution function for $T(k) \leq \tau$ is assumed to be exponential: $G(\tau) = 1 - \exp[-\lambda(k)\tau]$. It follows that the density $g(\tau) = G'(\tau) = \lambda(k)\exp[-\lambda(k)\tau]$. The expected value of the product k is therefore given by

$$\begin{aligned} E_T\, V(k;T) &= \Pi(k)\, \lambda(k)/r \int_o^\infty (1 - e^{-r\tau})\, e^{-\lambda(k)\tau} d\tau \\ &= \frac{\Pi(k)}{r + \lambda(k)} \end{aligned} \tag{3}$$

The incumbent firm is potentially replaced by an outside competitor producing a superior product[8] denoted by $k+1$. The expected reward for participating in the race for product $k+1$ per unit of time is

$$\frac{\lambda(k)\, \Pi(k+1)}{r + \lambda(k+1)}, \tag{4}$$

where $\lambda(k+1)$ is the probability per unit of time for the arrival of product $k+2$.

In order to preserve its dominance, the incumbent will carry out just sufficient research to deter outsiders to enter the R&D race; see Klette (1996a) for details. We assume that there is free entry into research and that there is a sunk cost associated with entering the race[9]. The zero profit condition requires that the net present value of participating in the race is equal to the sunk cost (F), i.e.[10]

$$\begin{aligned} &\int_0^\infty e^{-[r+\lambda(k)]t}[\lambda(k)EV(k+1) - R] \\ = &\ \frac{\lambda(k)\ EV(k+1) - R}{r + \lambda(k)} = F\,(R) \end{aligned} \tag{5}$$

The notation $F(R)$ reflects that sunk cost is allowed to depend on the level of the R&D activity. To be able to solve out for the optimal level of R&D, we must make an additional assumption about the functional form for the innovation-function $h(R,k)$:

Assumption 3: *Probability of project success and quality improvement per unit of time, is given by*

$$\lambda = R\,\phi(k). \tag{6}$$

[8]Notice that just a small set up cost for new entrants producing the current product will ensure that no entrants will enter the market with a product k, since the Bertrand competition will drive out all profits.

[9]As pointed out by Gilbert and Newbery (1984) in a similar analysis, a free entry equilibrium is only well defined in the presence of sunk cost in R&D. Sunk costs in R&D is also an essential part of Cohen and Klepper's (1996) analysis of R&D investment.

[10]This solution is the optimizing level for the incumbent if the profit margin and the innovative opportunities in the industry are moderate; see Klette (1996a) for a discussion.

Assuming constant returns with respect to R in the innovation function as in (6) is analytically very convenient, but empirically questionable. The functional form for the innovation function $h(R,k)$ is, of course, related to the issue of "diminishing returns in R&D" that has been the subject of much research on the basis of patent statistics; see Griliches (1990) for a survey. In a study of the relationship between patents and R&D in a panel of U.S. firms, Hall, Griliches and Hausman (1986) estimated the elasticity to between 0.3 and 0.6 in the longitudinal dimension, suggesting rather sharply diminishing returns. However, the appearance of diminishing returns in the longitudinal dimension could be an artifact due to the incompleteness of the underlying data rather than a reflection of the characteristics of the underlying innovation process (cf. Griliches, 1990).

Even if the estimated elasticity of patents with respect to R&D expenditures had been precisely estimated in the patent studies, it is not clear how that should be translated to form of the innovation function $h(R,k)$. That is, what is the relationship between a patent and making a step on the quality ladder; does a patent increase sales with a certain percentage or is the (percentage) increase in sales from a patent dependent on the stock of patents and the levels of sales. This is a (another) functional form question that remains open[11]. We will return to the discussion of the functional form for the innovation function $h(R,k)$ below.

The sunk cost in R&D, $F(R)$, is assumed to increase with the magnitude of the research effort; larger R&D activity requires e.g. more laboratory facilities and other setup costs. For simplicity, we will assume that the sunk cost increases linearly with the size of the R&D effort, i.e. $F(R) = \nu_0 R$. With this specification and (6), it follows from equation (5) that the incumbent's R&D effort is determined by

$$R = \frac{EV(k+1)}{\nu_0} - \frac{1}{\phi(k)}\left(\frac{1}{\nu_0} + r\right) \tag{7}$$

Inserting equation (3) and $\Pi(k) = \pi S(k)$ into equation (7), we get

$$R(k) = \frac{\pi S(k+1)}{\nu_0[r + \lambda(k+1)]} - \frac{1}{\phi(k)}\left(\frac{1}{\nu_0} + r\right) \tag{8}$$

[11] The relationship between sales and patents might be so noisy that it is hard to discriminate between alternative functional forms on the basis of the available data. The standard assumption in the literature is to assume a log-log relationship between patent counts and sales, rather than a linear-log relationship; see e.g. Griliches, Hall and Pakes (1991). Exploring the functional form of the relationship between patents and sales is an issue that we hope to examine in future research.

2.4 Firm growth according to Gibrat's law

We are now ready to show under what conditions the model will generate firm growth rates independent of firm size, according to Gibrat's law[12]. In order for Gibrat's law to hold in this model, we must have some diminishing returns in the innovation process. More precisely, we must assume that

Assumption 4: *Making product improvements gets more and more difficult, such that*

$$\phi(k) = \phi_0 \, e^{-\gamma k}. \tag{9}$$

with $\gamma \geq 0$.

The literature surveyed in Cohen and Klepper (1996) suggests that ϕ is a declining function of k (see Cohen and Klepper's Stylized fact 4). $\phi' < 0$ reflects "fishing out" effects where it gets harder and harder to make new innovations. This is another aspect of the "diminishing returns to R&D" issue that we also discussed above.

Some empirical evidence might be interpreted as support for assumption 4. The study by Bound et al. (1984) found that the number of patents per dollar of R&D is significantly lower for firms with larger R&D budgets[13]. Similarly, Acs and Audretsch (1991) found a negative relationship between innovations per R&D dollar and the level of R&D investment[14]. In the time series dimension at the aggregate level, several researchers have emphasized a related pattern; i.e. the increasing ratio of R&D per patent, see Caballero and Jaffe (1993), Kortum (1993) and Griliches (1994). One *possible* interpretation of these findings is that they reflect the property stated in assumption 4, cf. the discussion in Cohen and Klepper (1996).

On the other hand, both assumptions 3 and 4 on the functional form of the innovation function must be considered as tentative rather than as empirical facts at this stage[15]. At this stage, assumptions 3 and 4 are driven as much (or more) out of analytical convenience as from empirical support. We believe that further research is needed on this functional form issue, both research concerned with ways to alter or relax our assumptions in the theoretical analysis, and empirical research that tries to tie down the functional form of the innovation function[16].

[12] This corresponds to the steady state solution in the macro economic growth models.

[13] Griliches (1990) discusses alternative interpretations of this finding by Bound et al., and he argues that the pattern might be an artifact due to a sample selectivity problem for small firms.

[14] Acs and Audretsch's result is based on a cross sectional analysis of a comprehensive data set on innovations collected by the Small Business Administration.

[15] Another question that we have not discussed above, is whether it is correct to associate the longitudinal results as related to the R part in the $h(R, k)$-function, while the cross sectional results is associated with the k-part.

[16] See also Griliches (1994) for a broader argument reaching a similar conclusion.

Let us write $S(k) = \Psi\, e^{\gamma k}$, where Ψ captures all the other factors apart from the firm's own quality, that affect the firm's demand (as in section 2.2). It follows by inserting this expression and equation (9) into equation (8), that the R&D intensity can be written

$$\frac{R}{S} = \frac{e^{\gamma}\pi}{(r+\lambda)\nu_0} - \frac{1}{\phi_0\Psi}\left(\frac{1}{\nu_0} + r\right) \tag{10}$$

In the Appendix B, we have shown that λ will be independent of k in this model. This is due to two offsetting forces (cf. assumption 3); at the one hand, innovation is assumed to get harder and harder as spelled out in Assumption 4. On the other hand, the incentives to invest in R&D grow with the level of the firm's expected profits. Since future expected profits is proportional to the firm's current sales in our model, we have that the incentives to invest in R&D increases with the firm's sales, as shown in equation (8). With the functional form assumptions made above, the two effects just offset each other, with the result that λ turns out independent of k[17]. It follows that the whole right hand side of (10) is independent of k. Equation (10) has a number of implications, that we will examine next.

3 Main propositions derived from the model and the empirical evidence

3.1 Results related to firms within the same industry

Let us for the moment assume that differences in firm size is largely due to differences in product quality rather than differences in prices and production costs[18]. Then, since the right hand side of equation (10) does not involve k, we have that

Proposition 1 *R&D increases proportionally with firm size, and the R&D intensity is independent of sales.*

The empirical literature that relates to this issues is vast as is clear e.g. from the survey by Cohen and Klepper (1996)[19]. Let us consider the study by Bound et al. (1984) based on the Compustat file. Bound et al. concluded, after checking a number of econometric issues, that R&D increases proportionally to sales. There were deviations from this pattern among very large and very small firms, which tended to be more R&D intensive than the rest. However, as they point out, very small firms on the Compustat files are likely to be more innovative and do more

[17] A similar argument is made by Caballero and Jaffe (1993), Kortum (1993) and Griliches (1994).

[18] The reader should notice that there is a distinction between two dimensions of quality that we will discuss in section 4.3 below.

[19] See also the surveys by Cohen and Levin (1989) and Cohen (1995).

R&D than the average small firm in US manufacturing[20]. Cohen et al. (1987) confirmed this result at the firm level, and also at the line of business level for the sample of R&D performing business units. They report, however, some positive relationship between the R&D intensity and the size of the business unit for the sample of all business units. Klette and Griliches (1996) have found a similar pattern. In their survey, Cohen and Levin (1989) emphasize that the size effects in the R&D intensities, even if they are statistically significant, are "minute both in terms of the variance explained and the magnitude of the coefficients". Similarly, stylized fact 3 in Cohen and Klepper (1996) states that "in most industries it has not been possible to reject the null hypothesis that R&D varies proportionally with size across the entire firm size distribution". We conclude that Proposition 1, and hence the model presented in this paper, is a reasonable first approximation to at least one widely studied pattern of R&D investment.

To the extent differences in productions costs or prices are significant determinants of differences in firm sizes, proposition 1 needs to be qualified. Ψ, capturing the demand effect of differences in prices, appears on the right hand side of equation (10), and one can show that this factor will tend to create a positive relationship between the R&D intensity and firm size. If differences in prices can be captured by so-called "fixed effects", proposition 1 could be restated conditional on these fixed effects.

Equation (10) also implies that:

Proposition 2 *The R&D intensity is increasing with the profit margin.*

Two studies that empirically examine the relationship between R&D intensity and the profit margin are Geroski, Machin and van Reenen (1993) and Brouwer and Kleinknecht (1994). Both studies find a positive and statistically significant relationship. Brouwer and Kleinknecht's study is a pure cross sectional analysis, while the study by Geroski et al. uses a set of panel data. The study by Geroski et al. emphasizes a causal relationship that runs from changes in R&D to changes in the profit margin, but they also point out that the longitudinal effects are small compared to persistent cross sectional differences in the profit margins. In the model presented here, the causal relationship runs from profit margins to R&D; products with higher (expected) profit margins will *cet.par.* attract more R&D investment. Further research comparing the cross sectional and the time series relationship between profit margins and R&D might be able to clarify the causal relationship, but we recognize that it is notoriously hard to identify the causal structure by regression analysis. In section 3, we will discuss how to extend the model

[20] Griliches (1990) elaborates on this argument.

to accommodate higher profit margins for recent innovations and where the profit margins are gradually decreasing as the innovations get older. This extension allows for a causal relationship from R&D to higher profit margins, as identified by Geroski et al. If the extended model is correct, we should not be able to identify a one-way causal relationship between R&D and the profit margins, since the causality runs both ways.

We will now show that equation (10) implies Gibrat's law, i.e. firm sales independent of firm growth. Above we argued that λ is independent of k; see Appendix B for the formal argument. k determines the size (market share) of the firm if we neglect price differences (which we assume are unrelated to k; see the discussion in section 4 below). Since λ is the Poisson parameter in the stochastic process for k, we have established the following proposition:

Proposition 3 *Our model implies firm growth independent of firm size, in accordance with Gibrat's law.*

There is a long line of research on the empirical relationship between firm size and firm growth; see the survey by Sutton (1996). Recent studies include Evans (1987) and Hall (1987). Evans (1987) concludes that departures from Gibrat's law might be significant for small firms and long time periods, but one "might not go too wrong by maintaining Gibrat's law" for "short run changes in the growth and size distribution of the largest firms". Hall (1987) has carried out a careful analysis considering sampling bias and measurement errors, and she concludes that "Gibrat's law is weakly rejected for the smaller firms ... and accepted for the larger firms". The evidence suggests that Gibrat's law is not universally true, and it would not make sense to take that pattern for granted. However, we consider it desirable that our model can be made consistent with this benchmark case.

From propositions 1 and 3:

Proposition 4 *R&D follows a random walk (Gibrat's law in R&D).*

In a study of US manufacturing firms (over 8 years), Hall, Griliches and Hausman (1986) concluded that "R&D investment [is] essentially a random walk with an error variance which is small relative to the total variance of R&D expenditures between firms". The persistency of cross-sectional differences in R&D has also been documented in Klette and Johansen (1996).

It is well known that growth according to Gibrat's law will generate a skewed size distribution of firm size:

Proposition 5 *The model will generate a highly skewed distribution of firm size.*

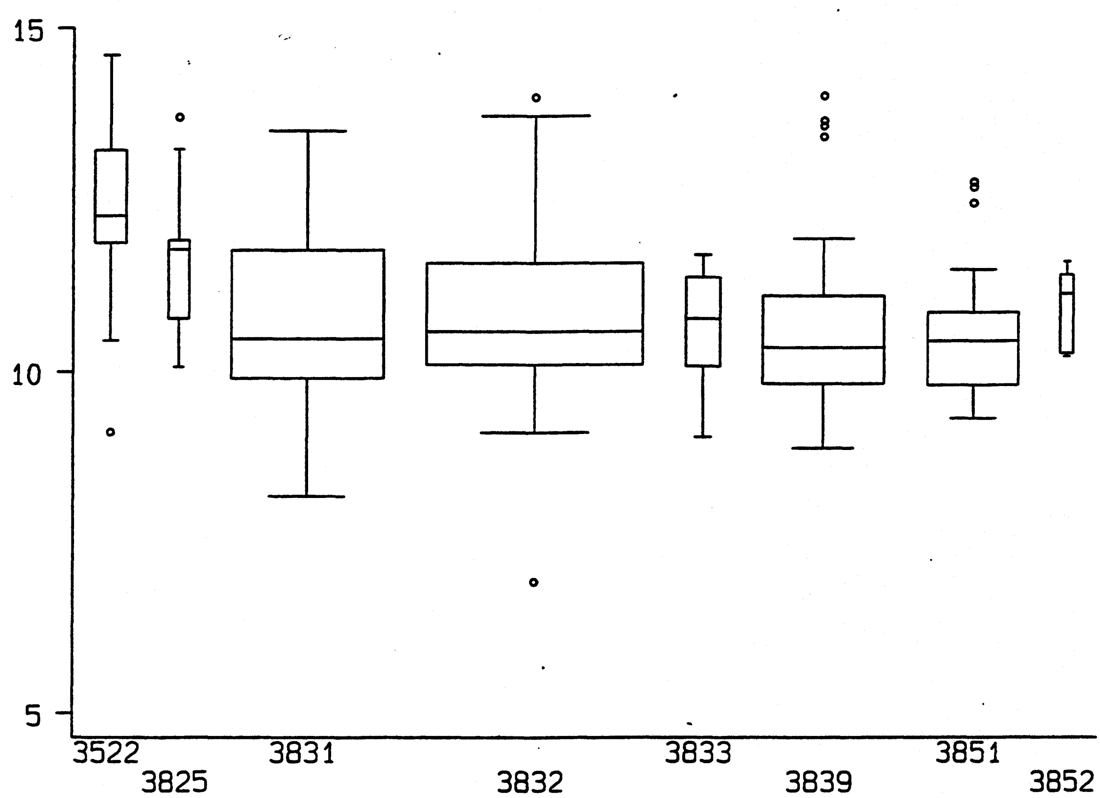

Figure 1: The size distribution for business units in the 8 most R&D intensive (4 digit ISIC) industries in Norwegian manufacturing. The width of the graph corresponds to the number of business units in the industry. The graphs are based on *log of sales for the business units*, and the boxes show the location of the quartiles in the distribution for each industry, together with the median shown as the horizontal line inside each box. The whiskers indicate the spread of the distribution in the lower and upper quartiles.

To our knowledge, there has been little systematic analysis of the skewness of the size distribution of high-tech industries (or other industries for that matter) since the book by Ijiri and Simon (1977). Figure 1 displays the size distribution for business units in the 8 most R&D intensive (4-digit) industries in Norwegian manufacturing. The width of the graphs in figure 1 corresponds to the number of business units in the industry. The graphs are based on *log of sales for the business units*, and the boxes show the location of the quartiles in the distribution for each industry, together with the median shown as the horizontal line inside each box. The whiskers indicate the spread of the distribution in the lower and upper quartiles. The graphs suggest that the distributions are even *more skewed than the log-normal distribution* in several industries, since the median is closer to the lower quartile than to the upper quartile, and the upper whisker is longer than the lower whisker.

Since the intensity parameter (λ) for the arrival of new innovations (i.e. increases in k) is independent of the level of k, the process will generate a Poisson distribution for k with a parameter which increases in proportion with the "age" of the process[21]. Since we consider the firms (in, say, an industry) to be different realizations of such processes, we have that:

Proposition 6 *The model will generate a more and more widely spread distribution of firm sizes when the average firm age increases.*

The increasing spread generated by Gibrat's law has been the focus of much research; see Steindl (1968) for a survey of the early attempts to create models for the firm size distribution, based on random walk growth but with a stable (non-increasing) spread of the distribution, and McCloughan (1995) for a recent study and more recent references. Entry and exit of firms can be one stabilizing force, as was early demonstrated by Simon and his coworkers; see Ijiri and Simon (1977). Notice that our model predicts an increasing spread of the firm size distribution only if the average firm age increases. The issue of average firm age is, of course, largely determined by the pattern of entry and exit, which is left open in the model presented above[22].

3.2 Differences in R&D intensities between industries

While entry (more precisely, preemption) within each product line is an endogenous part of our model, we do not explicitly model the entry and exit of competing product lines (cf. J in equation 1). It is consequently problematic to use our model, in particular equation (10), to explain differences in R&D between industries, as e.g. entry conditions typically differ across industries. More generally, the problem is that since industries differ, say, in terms of innovative opportunities, we would also expect Ψ (on the right hand side of equation 10) to differ.

If we, for argument's sake, abstract from this problem and treat Ψ as an independent parameter, we can use equation (10) to explain differences in R&D intensities between industries. The right hand side of equation (10) includes parameters related to innovative opportunities (ϕ_0), the consumer willingness to pay for higher quality products (γ) and other factors determining demand (Ψ). Equation (10) shows that, according to our model:

Proposition 7 *Industries with higher innovative opportunities, larger willingness to pay for higher quality products and larger demand will have higher R&D intensities.*

[21]The parameter in the Poisson distribution is proportional to (λt) in the case with a stationary intensity parameter (λ), where t is the age of the process. More generally, with a non-stationary intensity parameter, we have that the spread (and, of course, the mean) in the Poisson distribution is $\int_0^t \lambda(s)\,ds$.

[22]Pakes (1994) consider structural analysis of firm growth incorporating entry and exit decisions.

These results are intuitive and there is a large body of research supporting proposition 7; see the survey in Cohen (1995, section 4)[23]. Our model suggests how observables corresponding to these parameters could be incorporated in a structural analysis of R&D investment and firm growth across industries. We will add a few comments on this issue in the final section.

4 The demand side and price setting

4.1 Stronger firm interaction: Growth *and* decline of firms

The standard quality ladder model captures little in terms of competition between available products. This is our primary motivation for introducing an alternative specification of the demand side. In our specification, based on discrete choice theory for the demand of differentiated products, it is *relative product quality* that determines the firms' market shares, as well as *relative price*. With this specification a firm has to upgrade the quality of its product at the pace of its competitors in order to preserve its market share. This is clear from equation (1), which can be rewritten in terms of market shares, where the market share of a firm is given by

$$s_i = \frac{S_i}{M} = \frac{e^{v(k_i,P_i,\omega_i)}}{\sum_j e^{v(k_j,P_j,\omega_j)}} \tag{11}$$

with $v(k_i, P_i, \omega_i) = \gamma k_i - \alpha \ln P_i + \omega_i$[24]. In our model, a producer of a given (differentiated) product is facing competition from two different margins: (i) Competition from other differentiated products as captured by the denominator in equation (11), and (ii) competition from the lower quality versions of the same differentiated product. The case where the price determination of a new product is constrained by (ii) - competition from the lower quality versions of the same differentiated product - corresponds to a *non-drastic* innovation in Arrow's (1962) terminology, adopted in the patent race literature[25]. Otherwise the innovation is termed *drastic*.

4.2 Optimal price setting and the profit margin

Drastic innovations

In the case of drastic innovations, a firm with a new innovation k_i is not constrained in its price setting by the presence of lower quality varieties of the same product. The conditions for this

[23] See also Schmookler (1966) and Pakes and Schankerman (1984).

[24] This specification, although widely used, is quite restrictive (see e.g Berry, 1994, for a discussion). In particular, it might be natural to incorporate higher order terms that allows for interaction between the price elasticity (cf. α) and the product quality. Unfortunately, such an extension creates substantial analytical problems in the dynamic analysis.

[25] The terms drastic and non-drastic innovations are usually applied to situations with process innovations, but they are equally applicable to product innovations with the demand system specified as in our model.

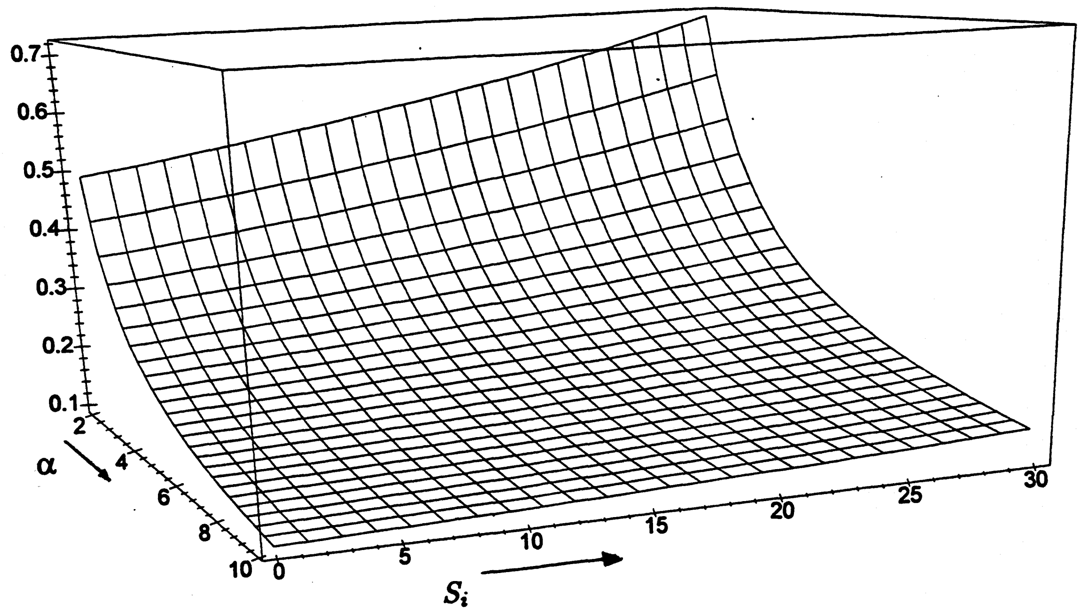

Figure 2: The profit margin (π) as a function of the market share (s_i), for given value of the price elasticity (α). Cf. equation. (14). (The vertical axis shows π, $\varepsilon = 1$).

to hold will be spelled out below. The price in the case with a drastic innovation is determined from the first order condition

$$s_i + (P_i - c_i)\frac{\partial s_i}{\partial P_i} = 0, \tag{12}$$

where c_i is the firm's marginal costs. With Bertrand competition, we have that $\partial s_i/\partial P_i = -\alpha s_i(1 - s_i)/P_i$, and hence

$$P_i = \left(1 - \frac{1}{\alpha(1 - s_i)}\right)^{-1} c_i \tag{13}$$

From the cost side we have the following relationship between average and marginal costs $C_i/Y_i = \varepsilon c_i$, where C_i and Y_i are total variable costs and output, respectively, and ε is the returns to scale parameter. Hence, equation. (13) can be rewritten $C_i/(P_iY_i) = \varepsilon\left(1 - [\alpha(1 - s_i)]^{-1}\right)$ or in terms of the profit margin

$$\pi_i = \frac{P_iY_i - C_i}{P_iY_i} = 1 - \varepsilon + \frac{\varepsilon}{\alpha(1 - s_i)} \tag{14}$$

Equation (14) can be confronted with empirical observations. In particular, the equation predicts a monotonic positive relationship between market shares, i.e. revenue, and the profit margin. Figure 2 shows how the profit margin, as expressed on the right hand side of (14), varies with the market share, for different values of α (the scale elasticity ε is assumed to be one).

Figure 2 reveals that for moderate market shares and not too small price elasticities (cf. α), the profit margin increases slowly with the firm's market share. As a (strong) simplification of this property, assumption 1.B states that the profit margin π_i is determined independently of k_i. We will discuss how to relax this assumption below.

Non-drastic innovations

Let us assume that the lower quality variety of product i (denoted $k_i - 1$) can be made available at price equal to marginal costs c_i. If the price determined by equation (13) fulfills the condition $\ln P_i > \ln c_i + \gamma/\alpha$, the new (high-quality) product will be outcompeted by the old variety at this price. This is the case of a non-drastic innovation. In this case, as shown by Arrow (1962), the optimal price is the so-called "limit price", determined such that the lower quality product is just competed out of the market, i.e.

$$\ln P_i = \ln c_i + \gamma/\alpha \tag{15}$$

minus a little epsilon. Clearly, this price is determined independently of k_i, and hence the profit margin is independent of k_i and therefore the size of the firm (and its market share). Hence, this case is also consistent with the assumption 1.B, that π_i is fixed.

We are now ready to state the following proposition, valid for non-drastic innovations and, in many cases, valid as an approximation also for drastic innovations:

Proposition 8 *The profit margin is (roughly) independent of market share.*

See Schmalensee (1989), Stylized facts 4.11 and 4.12 and the subsequent discussion for an attempt to summarize the empirical evidence on this issue. Griliches and Cockburn (1994) found that increased competition from new generic drugs in the pharmaceutical industry they considered did not seriously affect the price charged by the incumbent firm, only its market share.

Our model implies that the profit margin is independent of product quality. Metrick and Zeckhauser (1996) provide some empirical support for this claim, based on their analysis of mutual funds and automobile industries. There is, however, also some indirect counter evidence in the study by Geroski, Machin and van Reenen (1994), as discussed in relationship to Proposition 2.

To summarize, proposition 8 does not hold universally, and it suggests that our framework should be restricted to studies of firms that are not too large relative to the market they compete in.

Declining profit margins and the age of the product variety

Empirical studies, e.g. Mansfield, Schwartz and Wagner (1981), suggest that the profit margin might be high for a *new* high-quality product variety, but that imitation eliminates innovative rents within a few years. Similarly, Berndt, Griliches and Rosett (1993) found that the rate of price change for a product is related to the age of the product[26]. We will now show how the model can be extended to incorporate a negative relationship between the profit margin and the age of the innovation.

Available evidence suggests, as mentioned, that the profit margin declines gradually after a new product is introduced. This pattern can be captured in our model in the case of non-drastic innovations, if we assume that the marginal costs for the old variety is gradually declining, e.g. as the knowledge of how to produce the lower quality variety diffuses to low cost producers. A declining profit margin can also be explained in this way for innovations that are drastic when the new variety is introduced. If the production costs for the old, lower quality variety is (rapidly) declining, the pricing of products of drastic innovations might also turn out to be constrained by the competition of lower quality varieties some time after the new, high quality variety was introduced.

Formally we can assume that the profits decline with the age of the product variety according to $\Pi_0 \, e^{-\rho\tau}$, where τ is the time elapsed since the most recent version of the product was introduced[27]. With this slight respecification we find that equation (2) should be rewritten $V(k) = \Pi_0(k)\,(1 - e^{-(r+\rho)T(k)})/(r+\rho)$. It follows that equation (3) then will be replaced by

$$E_T \, V(k;T) = \frac{\Pi_0(k)}{r + \rho + \lambda(k)}, \tag{16}$$

where $\Pi_0(k)$ is the profits of the firm when the current product variety was first introduced. Extending the model in this way does not create any substantial changes to the analysis, and it allows for a causal relationship running from R&D to innovation to profit margins (cf. our discussion of Proposition 2). From the extended model, we can consequently state the following proposition:

Proposition 9 *Profit margin is independent of product quality, but the profit margin is a declining function of the age of the most recent product variety.*

[26]See also Kuznets (1930).

[27]It is not too difficult to handle the case where the profit margin (rather than total profits) declines exponentially over time, i.e. $\pi_0 \, e^{-\rho\tau}$, but the expressions are more complicated and will not be presented here.

The point is that a model consistent with this proposition can be developed using the analysis in section 2, but replacing equation (3) with equation (16).

4.3 R&D as a non-rival input and product quality

So far we have assumed that productions costs are independent of the product quality. That is, our model assumes that once a new product improvement is developed and introduced, no additional resources are needed to produce this improved product as compared to the older version. Romer (1990) has emphasized this aspect of R&D, by labeling R&D a *non-rival* input of production. A number of cases can surely be listed suggesting that R&D is a non-rival input to production, but Adams and Jaffe (1996) and Klette (1996b) have presented evidence suggesting that R&D is *not* a completely non-rival input at the firm level. This can be a question of the level of aggregation and R&D might still be a non-rival input at the business or product line level.

Our framework can be considered to involve two different dimensions of quality, one dimension which is explicit in our model, captured by the index k_i, and a second dimension which is hidden in our model. The second dimension of quality can be altered by spending *more resources per unit of production* – this quality aspect is a *rival* output dimension; it is related to the notion of a *rival* good in Romer's (1990) terminology. More horsepower and leather seats in cars are examples of quality differences in "rival" dimensions.

The first dimension – emphasized in this paper – is the *non-rival* aspect of quality; as soon as you know how to produce goods of a higher quality in the k_i-dimension, you do not have to spend more resources per unit of output for producing the higher quality product. A faster processor for computers can serve as an example of a non-rival improvement in quality.

We have implicitly buried the rival dimension in the measurement of output quantities and prices, and in the marginal costs. It is not surprising that the costs related to the rival part of product development and research, increases with sales. The more interesting part of our model is the non-rival aspect of product development and research, which distinguishes R&D from other factors of production.

5 Conclusions and remaining issues

We have presented a fully specified model of endogenous firm growth, where R&D and innovations are the engines of growth. The model is tightly specified and is based on a number of *ad hoc* assumptions. However, these assumptions are not arbitrary; they were introduced in order to rationalize a number of empirical regularities that have been established from empirical research on firm growth and innovation. We have also examined the empirical content of the assumptions and argued that they are good or acceptable approximations to the findings in (available) empirical research.

On the basis of our analysis, we have argued that our model is promising as a benchmark model to understand:

1. Why the size distribution of firms is highly skewed, with persistent heterogeneity.
2. How Gibrat's law can be reconciled with optimizing behavior.
3. Why R&D intensity is largely independent of size, even in cases where R&D is largely a non-rival input.

A major benefit of our model is that we are able to show how these three patterns are related.

We must, however, admit that the correspondence between the empirical literature and the empirical content of our model is not perfect. For instance, our model largely ignores imitation and other sources of quality changes. Empirically, it is widely observed that even in high tech industries there is a large fraction of firms reporting no R&D activity. These firms presumably survive by imitation[28]. Another aspect of the model is that it leaves considerable heterogeneity to be rationalized. For instance, we observe empirically large and persistent differences in R&D *intensities* between firms in most industries (also at a disaggregated industry level). The sources of differences in R&D intensities have been subject of much research[29], and our model provides some handles to capture such heterogeneity, such as differences in profit margins (i.e. the closeness of substitutes) and differences in innovative opportunities. Identifying and modeling these different sources for heterogeneity is a crucial step towards structural estimation of the parameters in the model.

[28] See e.g. Nelson (1988) for a discussion of this point.

[29] See Pakes and Schankerman (1984) and Cohen (1995).

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7 Appendix A: Previous literature

The specification of our model is inspired by the macro-models of endogenous growth, in particular the quality ladder model developed by Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992). Most versions of the quality ladder model have the property that each innovation is introduced by a new firm, while the producers of the old product varieties are driven out of the market. It is difficult to reconcile this property with the persistency of large firms that we tend to see in empirical data, at least at an annual frequency (see references below). Thompson (1996) has made the same criticism of the quality ladder growth model. He develops a complete model of R&D investment and firm growth, somewhat different from the model we present. On the basis of his model, he presents an empirical analysis of the relationship between R&D and the stock market value of the firm.

Our model is similar to the model presented in Barro and Sala-i-Martin (1995, ch.7) that emphasizes the persistent dominance of the established firms, rather than the continuous replacement of the leading firm. As shown in Klette (1996a), there is an error in the model by Barro and Sala-i-Martin (1995, ch. 7.4), and this is why we have developed our own model. Specifically, the second order condition associated with the first order condition in equation (7.39) is not satisfied. That is to say, the level of R&D investment Barro and Sala-i-Martin consider for the incumbent is not the optimal level of R&D investment.

Our (partial equilibrium) version of the quality ladder model is closely related to patent race models discussing the persistence of monopoly; see in particular Gilbert and Newbery (1982, 1984). Reinganum's (1985) model of industry evolution is also related to our analysis. Her model has the same property as the quality ladder models referred to above, where each innovation is introduced by a new firm. The model presented below extends the analysis by Reinganum and Gilbert-Newbery by allowing for competition between horizontally differentiated products. In our model each competing product (variety) is upgraded separately, i.e. each competing product is moving up its own quality ladder.

Our analysis has a similar focus as the work by Ericson and Pakes (1995), Pakes and Ericson (1990) and Pakes and McGuire (1994). They also present a fully specified model of firm growth with investment in innovative activities as the determinant of firm growth. Their model is more sophisticated than ours in that it allows for more extensive strategic interactions and considerations between the firms than we do. They also have a more complete analysis with respect to firm entry and exit. However, the cost of this sophistication is that the Ericson-Pakes model is analytically difficult to handle, and the model must be examined through simulations, while our simpler model is analytically tractable. The two models are to some extent complementary in the sense that our model can most easily be justified in situations where the firms considered are small relative to the market, while the case with many firms creates problems for the simulations of the Ericson-Pakes model (see Pakes and McGuire, 1994).

There are also some earlier studies of firm heterogeneity that are related to our study, such as Jovanovic (1982). (See also Lippman and Rummelt, 1982). Jovanovic's analysis is a dynamic version of Lucas' (1978) model, which is similar to the model by Kihlstrom and Laffont (1979). Both Lucas and Kihlstrom-Laffont present theoretical models that are consistent with heterogeneous firms, but they are both static and the sources of the heterogeneity are given exogenously, as is also the case for Jovanovic's model. The discussion of the active versus passive learning models in Pakes and Ericson (1990) is equally relevant for the relationship between Jovanovic's model and the model we present.

Sutton (1996) provides a comprehensive survey of theoretical and empirical studies of firm growth and firm heterogeneity.

Cohen and Klepper (1996) present a model that can rationalize a number of empirical regularities regarding R&D investment and firm size. Our model is in several respects similar to their analysis of the relationship between R&D and size, but it is somewhat more complete as it rationalizes why each firm's profits from its next innovation are constrained by its current size. More generally, our framework explicitly models firm growth, while firm growth is treated rather briefly in Cohen and Klepper's model. Klepper (1996) has also examined a related model of firm growth and industry evolution driven by innovation. Klepper's model is able to rationalize a number of interesting empirical regularities of firm growth and industry evolution, but the model is highly stylized at the individual firm level and therefore difficult to reconcile with structural estimation based on firms as the unit of observation.

Dasgupta (1986) is to some extent also related to our paper, in that he presents a theoretical model consistent with a number of the empirical observations we also consider (and some others). However, Dasgupta's analysis is based on a static model and does not make any predictions about patterns of firm growth and firm heterogeneity. Indeed, his model assumes identical firms. The theoretical literature on preemption and the persistence of incumbency in patent races, that we have already referred to above, is more relevant for our analysis (cf. the papers by Gilbert and Newbery, 1982, 1984).

8 Appendix B: Proposition 3; the formal argument

In this appendix, we will show formally that λ is independent of k. Combining assumption 4 with equation (10), it follows that

$$R(k) = e^{\gamma k}\left(\frac{\pi\Psi e^{\gamma}}{\nu_0(r+\lambda)} - \frac{1+r\nu_0}{\nu_0\phi_0}\right).$$

With $\phi(k) = \phi_0 e^{-\gamma k}$ and $\lambda = R\phi(k)$, it follows after some algebra that

$$\lambda = \frac{1}{2\nu_0}\left([4\nu_0\pi e^{\gamma}\Psi\phi_0 + 1]^{1/2} - 1 - 2r\nu_0\right),$$

which does not depend on k.

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