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On Nash equilibrium in prices in an oligopolistic market with demand characterized by a nested multinomial logit model and multiproduct firm as nest

Abstract:
This note provides a proof on existence and uniqueness of Nash equilibrium in prices in a market where the demand side is characterized by a nested multinomial logit model with multiproduct firm as nest and the supply side consists of oligopolistic price-setting multiproduct firms with each producing various differentiated variants.

Keywords: oligopolistic market, multiproduct firm, nested multinomial logit model, Nash equilibrium

JEL classification: C25, C62, C72, D43, L13

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1. Introduction

Recent years have seen a frequent use of random utility discrete choice models in demand and supply analysis for differentiated products under oligopolistic competition setting (e.g., Berry et al., 1995; Goldberg, 1995; Wojcik, 2000). Within this context, market demand is derived from discrete choice models of consumer behavior. The random utility of consumers depends on product attributes as well as individual characteristics; product market shares are then derived as the aggregate outcome of consumer decisions. On the supply side, firms are modeled as price-setting oligopolists, and endogenous market outcomes are derived from Nash equilibrium in prices.

For the sake of applying this framework for economic analysis, it is of interest to provide conditions for the existence and/or uniqueness of price equilibrium. Caplin and Nalebuff (1991) provide general conditions under which the proof of existence and uniqueness of pure strategy Nash price equilibrium can be established. Anderson et al. (1992) also prove that there exists a unique price equilibrium for a multinomial logit model. However, both results are for single product firms only.

In more realistic circumstances where multiproduct firms are involved, the existence and uniqueness of price equilibrium is usually assumed rather than proved *a priori* (e.g., Berry, 1994; Berry et al., 1995; Goldberg, 1995). Therefore, the purpose of this note is to make an extension of the results for single product firms to the case of multiproduct firms. Following the method of Anderson et al. (1992), we provide a proof on existence and uniqueness of Nash equilibrium in prices in a nested multinomial logit model with multiproduct firm as nest.

2. Model description

Consider m firms where firm j produces K_j variants of a differentiated product. Assume that firm j, $j = 1, 2, ..., m$, has fixed cost F_j and produces at constant marginal cost c_j^k for its variants k, $k = 1, 2, ..., K_j$.

There are N consumers in the economy and consumer i has utility

(1) $$U_{ij}^k = V_{ij}^k + \varepsilon_{ij}^k = y_i + \mathbf{Z}_j^k \boldsymbol{\beta} - w_j^k + \varepsilon_{ij}^k$$

for purchasing variant k produced by firm j.

Here y_i is consumer i's income; $\mathbf{Z}_j^k$ is a vector with components being product attributes other than price; $\boldsymbol{\beta}$ is a parameter vector and w_j^k is the price of variant k produced by firm j; $\{\varepsilon_{ij}^k\}$ are random error terms that are supposed to capture unobservable product attributes as well as unobservable individual-specific characteristics. The joint c.d.f. of the error terms is assumed to have the following multivariate extreme value distribution[1]

$$\text{(2)} \qquad \Pr\left(\bigcap_{j,k}\left(\varepsilon_{ij}^k \le x_j^k\right)\right) = \exp\left(-\sum_q \left(\sum_{r=1}^{K_q} \exp\left(-x_q^r/\mu_2\right)\right)^{\mu_2/\mu_1}\right),$$

where μ_1 and μ_2 are positive parameters, such that $\mu_1/\mu_2 \in (0,1]$, and have the interpretation that, $Corr\left(\varepsilon_{ij}^k, \varepsilon_{ij}^r\right) = 1 - (\mu_2/\mu_1)^2$ and $Var\left(\varepsilon_{ij}^k\right) = (\mu_1\pi)^2/6$. Moreover, (2) implies that $Corr\left(\varepsilon_{ij}^k, \varepsilon_{iq}^r\right) = 0$ when $q \neq j$.

Thus, μ_1 and μ_2 indicates the inter- and intra-firm heterogeneity, respectively. If $\mu_1 > \mu_2$, the variants within a firm are closer substitutes than those produced by other firms (Ben-Akiva and Lerman, 1985). If $\mu_1 = \mu_2$, all variants no matter where they are produced are equally "distant" in terms of difference, then the nested multinomial logit model will boil down to a multinomial logit model, as we shall see below.

Given the above setting, the choice probability of consumer i for choosing variant k produced by firm j, $P_j^k(\mathbf{w})$, equals (we suppress subscript i from now onwards for the sake of notational simplicity)

$$\text{(3)} \qquad P_j^k(\mathbf{w}) = \Pr\left(U_j^k = \max_{q \le m}\left(\max_{r \le K_q} U_q^r\right)\right) = \frac{\exp\left(S_j/\mu_1\right)}{\sum_{q=1}^{m}\exp\left(S_q/\mu_1\right)} \cdot \frac{\exp\left(\dfrac{\mathbf{Z}_j^k\boldsymbol{\beta} - w_j^k}{\mu_2}\right)}{\sum_{r=1}^{K_j}\exp\left(\dfrac{\mathbf{Z}_j^r\boldsymbol{\beta} - w_j^r}{\mu_2}\right)},$$

where $\mathbf{w}$ is a price vector for all products produced by all firms and

[1] For this type of distribution and the derivation of the associated nested multinomial logit model, see Ben-Akiva and Lerman (1985, p. 304-310).

(4) $$S_j = \mu_2 \ln \sum_{r=1}^{K_j} \exp\left(\frac{\mathbf{Z}_j^r \boldsymbol{\beta} - w_j^r}{\mu_2} \right).$$

Here S_j can be interpreted as the expected utility that consumer i receives from the choice among the products in firm (nest) j.

If we define $Q_j(\mathbf{w})$ as the marginal probability of choosing firm j and $R_j^k(\mathbf{w})$ as the conditional probability of choosing variant k given that k is produced by firm j, it follows from (2) that

(5) $$Q_j(\mathbf{w}) = \frac{\exp(S_j / \mu_1)}{\sum_{q=1}^{m} \exp(S_q / \mu_1)},$$

and

(6) $$R_j^k(\mathbf{w}) = \frac{\exp\left(\frac{\mathbf{Z}_j^k \boldsymbol{\beta} - w_j^k}{\mu_2} \right)}{\sum_{r=1}^{K_j} \exp\left(\frac{\mathbf{Z}_j^r \boldsymbol{\beta} - w_j^r}{\mu_2} \right)}.$$

Hence, we realize that $P_j^k(\mathbf{w})$ can also be written as

(7) $$P_j^k(\mathbf{w}) = Q_j(\mathbf{w}) \cdot R_j^k(\mathbf{w}).$$

3. Market Equilibrium

Assume that firm j takes the prices set by all other firms as given and it knows the mean demand $NP_j^k(\mathbf{w})$ for its variant k as a function of price vector $\mathbf{w}$. Consequently, firm j's decision problem is to choose the prices of all its variants, $w_j^1, w_j^2 \ldots w_j^{K_j}$, in order to maximize its expected profit π_j conditional on all other firms' prices, where profit is given as

(8) $$\pi_j = \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) NP_j^k(\mathbf{w}) - F_j.$$

Inserting $P_j^k(\mathbf{w})$ from (3) into (8) and maximizing (8) with respect to $w_j^1, w_j^2 \ldots w_j^{K_j}$ with the prices of other firms as given yields

Proposition 1:

Assume that consumers i ' utility function is given by (1) and (2). Under oligopolistic price competition, if market equilibrium exists, firm j 's equilibrium prices must satisfy the following equations:

$$(9) \qquad w_j^1 - c_j^1 = w_j^2 - c_j^2 = \ldots = w_j^{K_j} - c_j^{K_j} = \frac{\mu_1}{1 - \sum_{k=1}^{K_j} P_j^k(\mathbf{w})} = \frac{\mu_1}{1 - Q_j(\mathbf{w})}, \; j = 1, 2, \ldots, m .$$

The proof of Proposition 1 is given in the Appendix.

Proposition 1 states that at market equilibrium firm j will equalize the mark-up (difference between price and marginal cost) for each variant in order to maximize its total profit from all variants it produces. The fact that μ_2 does not appear in (9) indicates that at the equilibrium the intra-firm diversity doesn't matter.

4. A proof on existence and uniqueness of Nash equilibrium in prices

As shown, (9) provides the necessary first order conditions for the market equilibrium prices $w_j^1, w_j^2 \ldots w_j^{K_j}$ to maximize firm j 's expected profit π_j, taking other firms' prices as given. The sufficient conditions are guaranteed by the following proposition.

Proposition 2:

Taking other firms' prices as given, the market equilibrium prices derived from the first order conditions as given by (9), $w_j^1, w_j^2 \ldots w_j^{K_j}$, maximize firm j ' s expected profit π_j in (8).

The proof of Proposition 2 is given in the Appendix.

Clearly, $w_j^1, w_j^2 \ldots w_j^{K_j}$, as given by (9) are conditional on all other firms' prices; they are *de facto* firm j's best response function to all other firms' prices. The question of whether these (m) best response functions intersect at the same point in the price space, $\mathbf{w}^{NE}$, which gives the Nash price equilibrium, is answered by Proposition 3.

Proposition 3:

For the nested multinomial logit demand under oligopolistic price competition with each multiproduct firm as nest, there exists a unique Nash price equilibrium implicitly given by (9).

The proof of Proposition 3 is given in the Appendix.

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Appendix

Proof of *Proposition 1*:

The first order condition

$$\frac{\partial \pi_j}{\partial w_j^s} = 0 \quad \text{yields}$$

$$\frac{\partial \pi_j}{\partial w_j^s} = NP_j^s \left\{ 1 + \frac{1}{\mu_1} \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) P_j^k - \frac{1}{\mu_2} \left(w_j^s - c_j^s \right) + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) R_j^k \right\} = 0 \,,$$

for $s = 1, 2, ..., K_j$. Then (9) follows.

QED.

Proof of *Proposition 2*:

The second order conditions for maximizing firm j's expected profit π_j are given by

$$\begin{aligned} \frac{\partial^2 \pi_j}{\partial \left(w_j^s \right)^2} &= N \frac{\partial P_j^s}{\partial w_j^s} \left\{ 1 + \frac{1}{\mu_1} \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) P_j^k - \frac{1}{\mu_2} \left(w_j^s - c_j^s \right) + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) R_j^k \right\} \\ &+ NP_j^s \left\{ \partial \left(\frac{1}{\mu_1} \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) P_j^k \right) / \partial w_j^s - \frac{1}{\mu_2} + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) R_j^s + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) \frac{\partial R_j^k}{\partial w_j^s} \right\} \\ &= NQ_j R_j^s \left\{ -\frac{1}{\mu_2} + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) R_j^s \right\} < 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 \pi_j}{\partial w_j^s \partial w_j^t} = \frac{\partial^2 \pi_j}{\partial w_j^t \partial w_j^s} &= N \frac{\partial P_j^s}{\partial w_j^t} \left\{ 1 + \frac{1}{\mu_1} \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) P_j^k - \frac{1}{\mu_2} \left(w_j^s - c_j^s \right) + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) R_j^k \right\} \\ &+ NP_j^s \left\{ \partial \left(\frac{1}{\mu_1} \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) P_j^k \right) / \partial w_j^t + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) R_j^t + \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) \sum_{k=1}^{K_j} \left(w_j^k - c_j^k \right) \frac{\partial R_j^k}{\partial w_j^t} \right\} \\ &= NQ_j R_j^s \left\{ \left(\frac{1}{\mu_2} - \frac{1}{\mu_1} \right) R_j^t \right\} > 0, \end{aligned}$$

for $s, t \in \left\{ k : k = 1, 2 ... K_j \right\}$ and $s \neq t$, where the use is made of (9) and the fact that

$$\sum_{k=1}^{K_j} R_j^k = 1 \quad \text{and} \quad \sum_{k=1}^{K_j} \frac{\partial R_j^k}{\partial w_j^s} = 0\,,\ s = 1,2,...,K_j\,.$$

Consider the r^{th} order $\left(r = 1,2...K_j\right)$ leading principal minors of the Hessian matrix, $D_r\left(\mathbf{w}\right)$.

$$D_r\left(\mathbf{w}\right) = \begin{vmatrix} \dfrac{\partial^2 \pi_j}{\partial\left(w_j^1\right)^2} & \dfrac{\partial^2 \pi_j}{\partial w_j^1 \partial w_j^2} & \cdots & \dfrac{\partial^2 \pi_j}{\partial w_j^1 \partial w_j^r} \\ \dfrac{\partial^2 \pi_j}{\partial w_j^2 \partial w_j^1} & \dfrac{\partial^2 \pi_j}{\partial\left(w_j^2\right)^2} & \cdots & \dfrac{\partial^2 \pi_j}{\partial w_j^2 \partial w_j^r} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial^2 \pi_j}{\partial w_j^r \partial w_j^1} & \dfrac{\partial^2 \pi_j}{\partial w_j^r \partial w_j^2} & \cdots & \dfrac{\partial^2 \pi_j}{\partial\left(w_j^r\right)^2} \end{vmatrix}$$

$$= \left(NQ_j\right)^r \prod_{s=1}^{r} R_j^s \begin{vmatrix} -\dfrac{1}{\mu_2} + \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^1 & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^2 & \cdots & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^r \\ \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^1 & -\dfrac{1}{\mu_2} + \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^2 & \cdots & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^r \\ \cdots & \cdots & \cdots & \cdots \\ \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^1 & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^2 & \cdots & -\dfrac{1}{\mu_2} + \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^r \end{vmatrix}$$

$$= \left(NQ_j\right)^r \prod_{s=1}^{r} R_j^s \begin{vmatrix} -\dfrac{1}{\mu_2} + \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^1 & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^2 & \cdots & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^r \\ \dfrac{1}{\mu_2} & -\dfrac{1}{\mu_2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{1}{\mu_2} & 0 & \cdots & -\dfrac{1}{\mu_2} \end{vmatrix}$$

$$= \left(NQ_j\right)^r \prod_{s=1}^{r} R_j^s \begin{vmatrix} -\dfrac{1}{\mu_2} + \sum_{i=1}^{r}\left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^i & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^2 & \cdots & \left(\dfrac{1}{\mu_2} - \dfrac{1}{\mu_1}\right) R_j^r \\ 0 & -\dfrac{1}{\mu_2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\dfrac{1}{\mu_2} \end{vmatrix}$$

$$= \left(\left(NQ_j\right)^r \prod_{s=1}^{r} R_j^s\right)\left(-\frac{1}{\mu_2}\right)^{r-1} \left\{\frac{1}{\mu_2}\left(\sum_{i=1}^{r} R_j^i - 1\right) - \frac{1}{\mu_1}\sum_{i=1}^{r} R_j^i\right\}$$

where the third equality is obtained by adding (-1) × 1st row of the determinant to each of the rest (r-1) rows; the fourth equality is from adding successively the 2nd column, the 3rd column,…, the r th column to the 1st column; the last equality comes from the fact that the determinant now has the property that all its lower triangular elements are equal zero. Then it follows that

$$(-1)^r D_r(\mathbf{w}) > 0, \; r = 1, 2 \ldots K_j .$$

By Theorem 17.12 of Sydsæter and Hammond (1995, p. 639), Proposition 2 holds.

QED.

Proof of *Proposition 3*:

Let

(A.1) $$w_j^1 - c_j^1 = w_j^2 - c_j^2 = \ldots = w_j^{K_j} - c_j^{K_j} = \omega_j .$$

Then (4) can be written as

(A.2) $$S_j = \mu_2 \ln \sum_{r=1}^{K_j} \exp\left(\frac{\mathbf{Z}_j^r \boldsymbol{\beta} - c_j^r - \omega_j}{\mu_2} \right) = \mu_2 \ln \sum_{r=1}^{K_j} \exp\left(\frac{\mathbf{Z}_j^r \boldsymbol{\beta} - c_j^r}{\mu_2} \right) - \omega_j = b_j - \omega_j ,$$

where we have defined

(A.3) $$b_j = \mu_2 \ln \sum_{r=1}^{K_j} \exp\left(\frac{\mathbf{Z}_j^r \boldsymbol{\beta} - c_j^r}{\mu_2} \right).$$

Evidently, (5) can be rewritten as

(A.4) $$Q_j(\mathbf{w}) = \frac{\exp\left((b_j - \omega_j) / \mu_1 \right)}{\sum_{q=1}^{m} \exp\left((b_q - \omega_q) / \mu_1 \right)} = \frac{\exp(x_j)}{\sum_{q=1}^{m} \exp(x_q)} ,$$

where

(A.5) $$x_j = \left(b_j - \omega_j\right) / \mu_1 .$$

Then (9) becomes

(A.6) $$b_j - \mu_1 x_j = \frac{\mu_1}{1 - \dfrac{\exp x_j}{\sum_{q=1}^{m} \exp x_q}}, \quad j = 1, 2, ..., m \text{ .}$$

Anderson et al. (1992, p. 264-266) have demonstrated that there exists a unique solution x_j, $j = 1, 2, ..., m$ to (A.6). Then from (A.1) and (A.5) the unique $\mathbf{w}^{NE}$ consisting of m price vectors $\left(w_j^1, w_j^2, ..., w_j^{K_j}\right)$, $j = 1, 2, ..., m$, can be found.

QED.

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