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## Labor Supply of Retiring Couples and Heterogeneity in Household Decision-Making Structure

**Abstract:**

This paper analyzes the labor market participation behavior of retiring couples in Norway. To account for the unobserved heterogeneity in decision-making structure within the household, I formulate a mixed model by assuming there are two types of households, the cooperative type and the non-cooperative type. I assume that non-cooperative households behave according to a Stackelberg game with the male as the leader, while cooperative households engage in a cooperative bargaining process. The estimation results show that more than half of the households are of the non-cooperative type.

**Keywords:** household labor supply, retirement, unobserved heterogeneity, mixed model

**JEL classification:** D10, J26

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# 1. Introduction

Recent studies of retirement behavior have recognized the fact that labor force status and transitions of older married couples are correlated. It has been argued that it is not possible to understand the labor supply decision of one spouse without considering the behavior of the other (David M. Blau (1997), Alan L. Gustman and Thomas L. Steinmeier (2000), among others). In this paper, I develop a particular framework for modeling joint elderly labor supply behavior of a two-member household. In contrast to earlier studies, this framework allows for unobserved heterogeneity on household decision-making structure. I then apply it to analyze the retirement behavior of elderly couples in Norway when a new option (early retirement) becomes available to the husband.

Since a household consists of different individuals with possibly different preferences, situations in which household members have conflicting interests can arise. When analyzing household decision-making behavior, it is essential to correctly model how household members resolve their conflicts and reach a joint decision in these situations. However, the approaches taken in the literature differ substantially. No consensus has been reached as to which approach is best to describe the interaction between household members. For a survey of household behavior models, see Ted Bergstrom (1997), Richard Blundell and Thomas MaCurdy (1999), and Frederic Vermeulen (2002).

Samuelson (1956) and Gary Becker (1973, 1974) are the first among economists who recognize the fact that a household consists of different individuals. Since the 1970s, a large literature on decision-making behavior within households has emerged, mostly inspired by the pioneering work by Gary Becker on economics of marriage (for Becker's contribution in this field, see Shoshana Grossbard (2004)). Roughly, the literature can be divided into two strands, the cooperative and non-cooperative approaches. The cooperative approach emphasizes Pareto efficiency as the characteristic of household decision-making. This strand of models includes the traditional unitary model, cooperative bargaining models, and collective household models. Examples of cooperative models include the studies by Marilyn Manser and Murray Brown (1980), Marjorie B. McElroy and Mary J. Horney (1981), Patricia Apps and Ray Rees (1988), and Pierre-André Chiappori (1988). On the other hand, the non-cooperative approach assumes that a joint decision is the outcome of a non-cooperative game between members of the household. See for example, Peter Kooreman (1994), Shelly Lundberg and Robert A. Pollak (1994), Bridget Hiedemann (1998), and Gustman and Steinmeier (2000). More recently, Zhiqi Chen and Frances Woolley (2001) try to tie these two strands together by providing a theoretical analysis of household demands and intra-household resource allocation using the framework of a non-cooperative Nash-Cournot game as well as a cooperative bargaining game. To

shed some light on the question of which model is better for empirical applications, Erik Hernæs, Zhiyang Jia and Steinar Strøm (2001) compare the empirical performance of different models in both strands using Norwegian data, and find that the Stackelberg game with the male as the leader provides a better fit to the data than the Nash game and the unitary model.

The basic assumption in all of these studies is that the same ‘decision-making structure’ is assumed to hold for all households in the population. However, this may not necessarily be true. Households differ in members’ education levels, experiences, levels of affection and cultural backgrounds etc. They could also differ in decision-making structures. However, very few studies have been concerned with this type of heterogeneity problem.

The present paper makes a first attempt at dealing with this issue. I propose a mixed model that explicitly accounts for the unobserved heterogeneity in the household decision-making structure. In contrast to the usual assumption that all households follow the same decision-making structure, the model allows for the possibility that households are of two different types with respect to their decision structures: the cooperative type and the non-cooperative type. I assume that households belonging to the non-cooperative type (type NC households) behave according to a Stackelberg game with the male as the leader, while households of the cooperative type (type C households) engage in a cooperative bargaining process and are able to reach a Pareto efficient choice. The model provides a more general framework for modeling household behavior and takes both component models as special cases. The idea is somewhat similar to the preference segmentation model proposed by Wagner A. Kamakura and Gary J. Russell (1989) in the marketing literature. But they only deal with the ‘parameter heterogeneity’ in a single-agent choice problem. In other words, the decision units differ only in the parameter value of the utility functions. The model derived in their study is essentially a mixed logit model with a discrete support.

The empirical analysis is based on register data from Statistics Norway. I use the sample of couples in which the husband was eligible for the Norwegian Early Retirement Scheme (AFP) between 1994 and 1996, and the wife was not eligible. I find that the share of households that belong to the non-cooperative type is around 60 per cent.

The rest of the paper is organized as follows. In section 1, I derive the model and discuss the method of estimation. In section 2, I consider the econometric specification, while section 3 gives a basic description of data sources and the sample used in the analysis. Estimate results are given in section 4. Section 5 concludes.

## 2. Theoretical Framework

I analyze the labor market decisions of elderly couples in Norway when a new option (AFP early retirement) becomes available to husbands. The available choices for the husband are either to take early retirement ( $y_m = 1$ ) or to continue to work ( $y_m = 0$ ), while the wife's choices are either not to work ( $y_f = 1$ ) or to work ( $y_f = 0$ )<sup>1</sup>.

Both the husband and the wife are assumed to have their own utility functions, which are denoted as  $U_m$  and  $U_f$  respectively. Gary Burtless (1990) shows that, if the individual's utility function depends only on her own choice but not her spouse's choice, it is not necessary to model both members' decisions simultaneously. However, this condition can hardly be fulfilled in practice mainly for two reasons. Firstly, the labor market choice of one member can affect the financial rewards of the other member through specific tax and social security rules. Secondly, the preference of one member can be different with different labor market status of the other member, for example, due to the complementarity of leisure between spouses or the existence of joint consumption and public goods within the household. So in the following, the individual's utility is specified as a function of both her own decision and her spouse's decision.

Since the couple's utilities are not known with complete certainty, I apply the random utility model (RUM) framework of Daniel L. McFadden (1974) and Charles F. Manski (1977), and treat the individual utilities  $U_m$  and  $U_f$  as random variables. In particular, I assume that the utilities have the form:

$$(1) \quad \begin{aligned} U_m(y_m, y_f) &= v_m(y_m, y_f) + \varepsilon_m(y_m), \\ U_f(y_m, y_f) &= v_f(y_m, y_f) + \varepsilon_f(y_f), \end{aligned}$$

where  $(y_m, y_f)$  is the choice dummy vector for the household defined above.  $v_k(y_m, y_f)$   $k = m, f$  are the deterministic parts of the utility functions, and  $\varepsilon_k(\cdot)$  are random variables, which account for unobserved taste variations across individuals and alternatives or unobserved variables in the utility functions.

As mentioned earlier, household behavior models differ greatly on how to model the interplay between the husband and the wife. In this paper, instead of assuming that all households follow the exact same structure, I make a less restrictive assumption that two types of households

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<sup>1</sup> In this study, working hours are exogenously determined and are not part of the choice alternatives. For wives, since they are not eligible to either the Early Retirement or Ordinary Retirement scheme in the sample, a withdraw from the labor force is characterized with zero labor income with full leisure, and is thus labeled as 'not to work'.

coexist. Namely, I assume that the households are composed of cooperative types and non-cooperative types. Each type is characterized by its decision-making structure.

## 2.1. Cooperative type: Households Following the Collective Model

Traditionally, empirical studies of household behavior have been based on the ‘first generation’ economic model that treats the household as a single decision-making unit. The model is often referred to as the ‘unitary model’ or ‘common preference model’ in the literature. According to this model, the household maximizes a joint utility function, or more precisely, a household welfare function subject to a pooled budget constraint. The most important characteristic of the unitary model, as well as other models of the cooperative type, is that the model implies that the household behavior is Pareto optimal. No household member can be made better off without making the other worse off. In the last two decades, there has been an increasing interest in refining the cooperative models by modeling household behavior as the result of a bargaining process<sup>2</sup>. Recently, Pierre-André Chiappori (1988,1992) proposed the collective approach to household behavior, where he only assumes that household decisions are Pareto efficient without imposing a particular bargaining setting<sup>3</sup>.

Similar to studies by Nicole Maestas (2001), Mauro Mastrogiacomo, Rob Alessie and Maarten Lindeboom (2004) and Zhiyang Jia (2004), I model the household's labor supply decision as an outcome of a cooperative bargaining process in the spirit of the collective approach. The household is assumed to maximize a collective household utility function  $U$  with respect to the decision variable  $(y_m, y_f)$ . The collective utility function is a weighted sum of the individual utilities of the husband and the wife:

$$(2) \quad U(y_m, y_f) = wU_m(y_m, y_f) + (1 - w)U_f(y_m, y_f).$$

Following Maestas (2001), Martin Browning and Pierre-André Chiappori (1998),  $w$  measures the husband's decision-making control in the household. If  $w = 1$ , the household behaves as if the husband has exclusive decision-making control, whereas  $w = 0$  implies that the wife has exclusive control. In this study,  $w$  is assumed to be constant over time for the same couple, and is not affected by the labor market decision.

The couple is assumed to choose the alternative  $(y_m, y_f)$  that gives the highest utility. However, since the utilities are random for analysts, I can only define the choice probability of choosing alternative  $(y_m, y_f)$  given that the household is of the cooperative type:

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<sup>2</sup> The household bargaining models are pioneered by Manser and Brown (1980) and McElroy and Horney (1981).

<sup>3</sup> A good survey on the collective household approach is Vermeulen (2002).

$$(3) \quad P_C(y_m, y_f) = \text{Prob}(U(y_m, y_f) = \max_{(y'_m, y'_f) \in \Omega} U(y'_m, y'_f))$$

where  $\Omega = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$  is the set of choices available to the household.

## 2.2. Non-Cooperative Type: Households Playing a Stackelberg Game

Although some consider the use of the non-cooperative game theoretical models in a household context as controversial, the empirical evidence does not always support the idea that household members always can reach a Pareto optimal decision. See for example, the study on risk sharing within households by Stefan Dercon and Pramila Krishnan (2000) and the study on intra-household production decisions by Chris Udry (1996)<sup>4</sup>.

In a non-cooperative framework, husband and wife are assumed to engage in a non-cooperative game to maximize their own utilities. The great advantage of these models is that the equilibrium is self-enforcing so that nobody will gain by deviating from the equilibrium. So in contrast to the cooperative models, non-cooperative models do not assume that husband and wife can enter binding, costless enforceable agreements, as pointed out by Lundberg and Pollak (1994) and Chen and Woolley (2001).

For households of the non-cooperative type (type NC households), I model the choice problem as a non-cooperative game. The player of the game, husband and wife, can take one of two actions, working or retirement/not working. The pay-off of the game is simply the utility function:  $U_k(y_m, y_f)$  defined in (1).

The pay-off matrix of the non-cooperative game is given in Table 1.

**Table 1. The pay-off matrix of the non-cooperative game**

Husband	Wife	
	Works, $y_f=0$	Home, $y_f=1$
Works, $y_m=0$	$U_m(0,0), U_f(0,0)$	1. $U_m(0,1), U_f(0,1)$
Retired, $y_m=1$	$U_m(1,0), U_f(1,0)$	2. $U_m(1,1), U_f(1,1)$

As in Hiedemann (1998), I assume that the roles of the husband and the wife in this game are asymmetric. The husband is assumed to be the leader, while the wife acts as a follower. That is, the choice behavior is modeled as a Stackelberg game with the male as the leader. Kooreman (1994) has developed a probability model to specify the probability of choosing each alternative for such a game.

<sup>4</sup> A review of arguments in favor of the assumptions that household members may behave non-cooperatively can be found in Kai A. Konrad and Kjell E. Lommerud (1995).

In the Appendix, I give a detailed discussion of this model, and re-derive the probability of the couple choosing each alternative  $(y_m, y_f)$  given that the household is of the non-cooperative type,

$$P_{NC}(y_m, y_f).$$

### 2.3. A mixed Model of Household Types

When there are externalities associated with the other member's retirement choice, households with the same socio-economic characteristics but of different types can behave differently. Obviously, applying the cooperative model on households of the non-cooperative type is wrong and results in misleading inferences. To deal with the problem that the household types are not directly observable, I assume that the data set is a random sample from a mixed population of the two types of households with unknown population share  $\lambda_s$ ,  $s = C, NC$  ( $0 \leq \lambda_s \leq 1$ , and  $\lambda_C + \lambda_{NC} = 1$ ).

Under this assumption, the unconditional probability of choosing state  $(y_m, y_f)$ ,

$P(y_m, y_f)$  is equal to:

$$(4) \quad P(y_m, y_f) = \lambda_C P_C(y_m, y_f) + \lambda_{NC} P_{NC}(y_m, y_f).$$

Where  $P_s(y_m, y_f)$  ( $s=C$  or  $NC$ ) is the conditional probability of choosing alternative  $(y_m, y_f)$  given the household type  $s$  specified earlier.

The new model is a mixture of the collective model and the Stackelberg model and a generalization of both.

The (incomplete) log-likelihood function can then be defined as follows:

$$(5) \quad \ln l = \sum_i (y_m^i y_f^i \ln P^i(1,1) + y_m^i (1 - y_f^i) \ln P^i(1,0) + (1 - y_m^i) y_f^i \ln P^i(0,1) + (1 - y_m^i)(1 - y_f^i) \ln P^i(0,0)).$$

Where the superscript  $i$  indexes the households and  $P^i(y_m, y_f)$  denotes the probability for household  $i$  to choose state  $(y_m, y_f)$ .

### 2.4. A Discussion of Identification of the Mixed Model in Principal

The mixed model developed above is not nonparametrically identified. Unlike standard discrete choice models, identification of the mixed model requires the specification of the latent classes (types of households in this analysis) first. The specification of the latent classes has to be based on economic

theory and/or earlier research. I consider it as a behavioral assumption and a part of the theoretical model rather than a part of empirical specifications that are required for identification.

Given the specification of the latent classes, parametric identification can be achieved through ordinary functional forms and distributional assumptions of the utility functions (1). Following an early result on identification of finite mixture by Sidney J. Yakowitz and John D. Spragins (1968), the mixed model is identified if and only if different latent classes provide different behavior predictions for some decision units for all possible values of the parameters to be estimated. In this study, the underlying probability generating structures  $P_s(y_m, y_f)$  for different classes have to be different; otherwise it is not necessary to distinguish them. Moreover, in practice, the number of the decision units is normally much larger than the number of unknown parameters. Given these conditions, it is impossible for  $P_C(y_m, y_f) = P_{NC}(y_m, y_f)$  to hold for all the decision units. So, Yakowitz and Spragins's (1968) identification condition imposes no real constraint in my model.

Similar to other models that are not nonparametrically identified, the mixed model may be highly sensitive to the specified parametric structures. Robustness check is normally needed. I will provide such a check on the parametric functional form assumptions later.

### 3. The Empirical Specification

#### 3.1. The Deterministic Part of the Utility Function

I assume that the deterministic parts of the utility functions (1) are defined as follows:

$$(6) \quad \begin{aligned} v_m(y_m, y_f) &= a_m \ln(C_{y_m, y_f}^m) + b_m \ln(L_{m, y_m}) + b_{mc} \ln(L_{y_m, y_f}), \\ v_f(y_m, y_f) &= a_f \ln(C_{y_m, y_f}^f) + b_f \ln(L_{f, y_f}) + b_{fc} \ln(L_{y_m, y_f}). \end{aligned}$$

Where  $C_{y_m, y_f}^k$   $k = m, f$  denotes the individual consumption given the decision choices  $(y_m, y_f)$ .

$L_{k, y_k}$   $k=f, m$  is the individual leisure, and  $L_{y_m, y_f}$  is the shared leisure term of the household.

According to this specification,  $a_m, a_f$  measure the contribution of economic incentives to the utility.

The  $b$  parameters measure the utility of leisure.

Since there is no direct observation on individual consumption values, I assume that there exists an income sharing rule  $C_{ij}^m = \mu_m C_{ij}$  and  $C_{ij}^f = \mu_f C_{ij}$  where  $C_{ij}$  is the household's disposable income.  $\mu_m, \mu_f$  lie between 0 and 1.  $\mu_m + \mu_f$  can be greater than 1 due to the existence of public goods. I assume here that the income-sharing rule is determined before the choice problem is

presented and is not according to the relative income level within the household, so it can be seen as exogenous. For different households, this sharing rule may be different, i.e. the parameter value  $\mu_m, \mu_f$  can be different across households. Inserting the income sharing rules into (6), we have

$$(7) \quad \begin{aligned} v_m(y_m, y_f) &= \gamma_m + a_m \ln(C_{y_m, y_f}) + b_m \ln(L_{m, y_m}) + b_{mc} \ln(L_{y_m, y_f}), \\ v_f(y_m, y_f) &= \gamma_f + a_f \ln(C_{y_m, y_f}) + b_f \ln(L_{f, y_f}) + b_{fc} \ln(L_{y_m, y_f}). \end{aligned}$$

Where  $\gamma_k = a_k \ln \mu_k$   $k=m, f$ . So, the income sharing parameters are transformed into the constant term of the utility functions. Recall that in a discrete choice setting it is only the difference in utility that matters, while the common factor in utilities of different alternatives is eliminated. So given the specification of the utility function forms, it is not possible to identify the sharing parameters.

The joint disposable income,  $C_{y_m, y_f}$ , is equal to annual after-tax income when the husband is in state  $y_m$  and the wife is in state  $y_f$ . It is defined as

$$(8) \quad C_{y_m, y_f} = r_{m, y_m} + r_{f, y_f} - T(r_{m, y_m}, r_{f, y_f}),$$

where  $r_{m, y_m}$  is the gross income of the husband when he is in state  $y_m$ , and  $r_{f, y_f}$  is the gross income of the wife when she is in state  $y_f$ , and  $T(\cdot)$  is the tax function. The unit of tax calculation is the couple, not the individual, which means that the taxes paid by the couple depend on the labor market states of both members in the household. All details of the tax rules have been accounted for when I construct the data. For both the husband and the wife, I allow the marginal utility w.r.t joint income to depend on the net wealth of the household, namely,

$$(9) \quad a_k = a_{k0} + a_{k1}(\text{household wealth}),$$

for  $k=f, m$ , since I suspect that households with different wealth will value disposable income differently.

$L_{k, y_k}$   $k=f, m$  is the individual leisure. It is defined as one minus the ratio of hours of work to total annual hours. For instance, for the case when the husband chooses to continue to work,  $L_{m, 0} = 1 - (37.5 * 46) / 8760$ . The common belief is that health condition plays an important role in the retirement decision. To test this hypothesis, I let the marginal utility w.r.t. male leisure depend on an indicator of husband's health: the ratio of sick leave to total working hours in the 15 months prior to early retirement eligibility (AFP-eligibility). That is

$$(10) \quad b_m = b_{m0} + b_{m1}(\text{sick history}).$$

Unfortunately, there is no similar information available for females in the dataset.

Following Yuk-fai Fong and Junsen Zhang (2001), I include the shared leisure term  $L_{y_m, y_f}$  in the utility function. It is defined as the leisure time that spouses spend together<sup>5</sup>. Shared leisure is used to account for the hypothesis that the household members not only derive utility from each member's individual leisure, but also from the leisure they enjoy together. If the parameter in front of the shared leisure term turns out to be positive, it partly explains the 'coordination' between husband's and wife's labor supply behavior — they tend to stop working at the same time, a pattern noticed by Michael D. Hurd (1997). Ricardo A. Godoy (2002) reports that the relation between shared leisure and spouse's age resembles a parabola on both US and Honduras data. To check whether there is a similar pattern in the data, I specify the marginal utility with respect to shared leisure as a quadratic function of the age difference (husband's age - wife's age):

$$(11) \quad b_{kc} = b_{kc0} + b_{kc1}(\text{agediff}) + b_{kc2}(\text{agediff})^2,$$

for  $k=f, m$ . To cope with the problem that I don't have a detailed time schedule for each of the spouses, I follow the study by Zhiyang Jia (2000) and use  $\min(L_{m, y_m}, L_{f, y_f})$  as a proxy for shared leisure in this analysis.

### 3.2. The Choice Probabilities of the Cooperative Households

As discussed in section 1, I assume that the cooperative households make their choices to maximize a weighted household utility function (2). Given the setting in this paper, similar to Mastrogiacomio, Alessie and Lindeboom (2004) and Jia (2004), it is not possible to identify the bargaining parameter  $w$  separately from the other parameters in the collective model. Only the parameters in the joint household utility  $U = wU_m + (1-w)U_f$  can be estimated. In fact, using (2) and (7), the collective household utility function can be rewritten as

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<sup>5</sup> Fong and Zhang (2001) use a different name 'spousal leisure' for the same concept. They argue that it is important to take into account 'spousal leisure' when considering the house-hold labor supply problem, and extend the standard collective household model to include 'spousal leisure'.

$$\begin{aligned}
(12) \quad U(y_m, y_f) &= w\gamma_m + (1-w)\gamma_f + (wa_m + (1-w)a_f)\ln(C_{y_m, y_f}) + wb_m \ln(L_{m, y_m}) \\
&\quad + (1-w)b_f \ln(L_{f, y_f}) + (wb_{mc} + (1-w)b_{fc})\ln(L_{y_m, y_f}) + (w\varepsilon_m(y_m) + (1-w)\varepsilon_f(y_f)) \\
&= \gamma + \alpha \ln(C_{y_m, y_f}) + \beta_m \ln(L_{m, y_m}) + \beta_f \ln(L_{f, y_f}) + \beta_c \ln(L_{y_m, y_f}) + e(y_m, y_f) \\
&= v(y_m, y_f) + e(y_m, y_f).
\end{aligned}$$

Where the deterministic part of the joint household utility function is defined as:

$$(13) \quad v(y_m, y_f) = \gamma + \alpha \ln(C_{y_m, y_f}) + \beta_m \ln(L_{m, y_m}) + \beta_f \ln(L_{f, y_f}) + \beta_c \ln(L_{y_m, y_f}).$$

Note that for both the husband and the wife, the marginal utility of household income depends on household wealth linearly as in (9). So  $\alpha$  in the joint utility function (13) is also a linear function of household wealth and can be rewritten as

$$(14) \quad \alpha = \alpha_0 + \alpha_1(\text{household wealth}).$$

Similarly, using (10) and (11), we have  $\beta_m = \beta_{m0} + \beta_{m1}(\text{sick history})$ ,

$$\beta_c = \beta_{c0} + \beta_{c1}(\text{agediff}) + \beta_{c2}(\text{agediff})^2.$$

Since I can only identify the joint preference parameters but not the individual preference parameters and the bargaining parameter  $w$ , the model above cannot be distinguished from the so-called "unitary model" where the household is treated as a single decision-making unit.

For the joint error terms  $e = w\varepsilon_m + (1-w)\varepsilon_f$ , I assume that they are *i.i.d.* standard extreme value distributed across choices. Then the choice probabilities have a multinomial logit structure as follows:

$$(15) \quad P_C(y_m, y_f) = \frac{e^{v(y_m, y_f)}}{\sum e^{v(y'_m, y'_f)}}.$$

### 3.3. The Choice Probabilities of the Non-Cooperative Households

The deterministic parts of the individual utility functions have been specified in (7). To derive the choice probabilities, it remains to specify the distribution of the error terms. We assume that  $\varepsilon_m(1), \varepsilon_m(0), \varepsilon_f(1), \varepsilon_f(0)$  is 4-variate normal distributed with zero mean and covariance matrix  $\Sigma$ , which is defined as:

$$\Sigma = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\rho}{2} & 0 \\ & \frac{\sqrt{2}}{2} & 0 & \frac{\rho}{2} \\ & & \frac{\sqrt{2}}{2} & 0 \\ & & & \frac{\sqrt{2}}{2} \end{pmatrix}$$

The motivation is that I wish to allow the possibility of any ‘common taste’ (correlation between the same choices) between husband and wife. The covariance matrix is specified so that the random variables of interest (as described in the Appendix)  $e_m$  and  $e_f$  are bivariate normally

distributed with zero mean and covariance matrix  $\begin{pmatrix} 1 & \rho \\ & 1 \end{pmatrix}$ . Using Table A.1 in the Appendix, the

choice probability for each state  $(y_m, y_f)$  can be specified. For example, the probability of both husband and wife choosing to work is given by:

$$(16) \quad \Pr(0,0) = \Pr(e_f < \min(v_f(0,0) - v_f(0,1), v_f(1,0) - v_f(1,1)) \text{ and } e_m < v_m(0,1) - v_m(1,0)) \\ + \Pr(v_f(1,0) - v_f(1,1) < e_f < v_f(0,0) - v_f(0,1), \text{ and } e_m < v_m(0,0) - v_m(1,1))$$

Detailed formulas are also provided in the Appendix.

## 4. Data

The data set is based on administrative registers from Statistics Norway. It contains detailed information on labor market behavior, income and other socio-economic variables at the individual level for virtually the entire working population. Information on marriage status, which allows us to identify the household, is also available.

For the present study, we use data from the period 1994-1996. During the observation period, 50 per cent of earnings in excess of the basic amount in the public pension system (USD 5 600) when retired were deducted from the pension. With a marginal tax rate on earnings and pensions at say 40 per cent, the effective tax rate on earnings was 70 per cent. So disregarding the option of combining earnings and early (partial) retirement in the choice set is not unreasonable.

I restrict the sample in this study to comprise all married couples in which the husband is eligible for AFP during the period from 1 October 1994 until 31 December 1996. This gives us a sample of 12,475 couples. Since the eligibility age was 64 from 1 October 1993 until 1 October 1997, the couples in the sample then knew at least one year in advance that retirement would become possible, and could plan retirement. I then restrict the data to couples in which the wife was between

50 and 67 years old and not eligible for AFP. These restrictions are imposed in order to make sure that the options postulated for the two spouses are reasonable. The restrictions reduce the sample down to 10,008 couples that fulfill all the criteria.

The dependent variables in this analysis are the labor supply choices of both household members  $(y_m, y_f)$ . Table 2 gives the frequency of each outcome.

**Table 2. Frequency of each outcome**

	Wife Works, $y_f=0$	Wife at Home, $y_f=1$
Husband Works, $(y_m=0)$	3693 (36.9%)	2712 (27.1%)
Husband Retires, $(y_m=1)$	2019 (20.2%)	1785 (17.8%)

Because the states are mutually exclusive, individuals' gross income can be observed only for one state. In order to model other possible outcomes, I need to impute or simulate the gross income for those states in which the individual is not observed. The following imputation rules are used. If the individual is not working in the year of analysis but was working in the previous year, then working state is characterized by the observed earnings from the previous year. This approach is more accurate than a standard wage regression, because the wage income is fairly stable for workers who are over 50 years old. If the wife is observed to be out of the labor force both the current and the previous period, then working income is imputed using a log-earning regression. Since the detailed earnings' history back to 1967 is available, potential AFP pension benefits are calculated very precisely using detailed pension rules. All the income variables are net of taxes using detailed tax rules with all the spousal characteristics considered. Some descriptive statistics after the imputation are given in Table 3.

**Table 3. Descriptive statistics**

Variable	Average	Min	Max
Household disposable income, when both are working (10,000 USD)	4.32	1.62	37.52
Household disposable income, when husband is working but wife is not (10,000 USD)	2.61	0.72	32.92
Household disposable income, when wife is working but husband is not (10,000 USD)	3.50	1.57	11.34
Household disposable income, when husband takes early retirement and wife is not working (10,000 USD)	1.78	0.95	2.34
Wealth (10,000 USD)	8.27	0	2742
Age difference (age of husband – age of wife)	3.6	-3	14
Sickness history (proportion of days on sick leave in the previous 15 months)	0.023	0	0.87
No. of Observations: 10,008			

Note: The average household disposable income for Norwegian households is around 34,000 USD in 1995. Source of the data: Unpublished administrative registers, Statistics Norway.

## 5. Estimation Results and Robustness Check

### 5.1. Estimation Results

To estimate the model, I maximize the log-likelihood function (5) with respect to the unknown parameters<sup>6</sup>. The parameters to be estimated include: both husband and wife's utility parameters in the Stackelberg game case, the joint utility function parameters in the collective model, and the share of each type (component weights)  $\lambda_s$ . Since  $\lambda_s$  lies within the interval  $[0,1]$ , a transformation

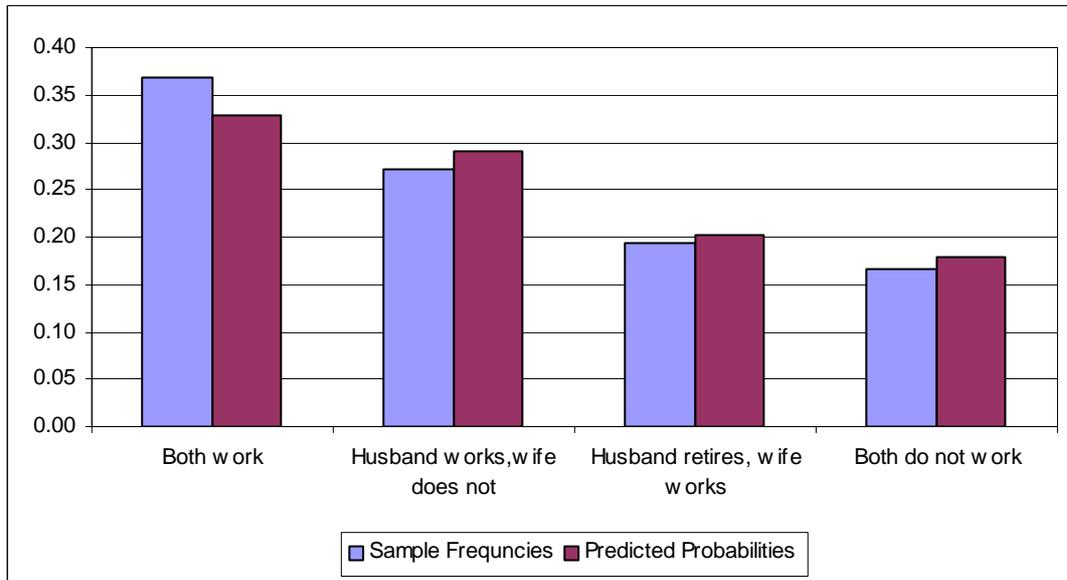
$\lambda_{NC} = 1/(1 + \exp(\delta))$  is used in the estimation instead of directly estimating  $\lambda_{NC}$ .

The overall fitting of the model is satisfactory, with the McFadden  $R^2$  to be around 15%. The model predicts the observed frequencies rather well. Figure 1 shows the observed and predicted probabilities across the different states.

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<sup>6</sup> A common practice when estimating a finite mixture model is to use the Expectation–Maximization Algorithm (EM algorithm, see (Arthur P. Dempster, Nan M. Laird and Donald B. Rubin, 1977)) as in James Heckman and Burton Singer (1984). The big advantage of this algorithm is its stability and easier computation compared with directly maximizing the likelihood function. However, for the present problem, I am not able to simplify the computation much by using the EM algorithm because the maximization step cannot be solved analytically, and the convergence turns out to be extremely slow. So I still choose to maximize the likelihood function directly.

**Figure 1. Predicted probabilities and sample frequencies**



The detailed estimation results are given in Table 4<sup>7</sup>. The share parameter is sharply determined, and around 61.7% of the households in the sample behave according to the Stackelberg game with the husband as the leader and are type NC households, while the rest behave according to the collective model and are type C households. This suggests that the traditional homogeneity assumption is not consistent with the data.

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<sup>7</sup> Due to some numerical problems, I have assumed that the marginal utility of shared leisure is a constant for both husband and wife in a non-cooperative household.

**Table 4. Maximum likelihood estimates for the mixed model**

Coefficient	Variable	Estimates	Z statistics
<b>Collective Model (cooperative type)</b>			
$\alpha_0$	Household disposable income: constant	7.691	24.0
$\alpha_1$	Household disposable income: linear in wealth	-0.024	-4.8
$\beta_{f0}$	Female leisure	1.897	1.2
$\beta_{m0}$	Male leisure: constant	1.342	3.0
$\beta_{m1}$	Male leisure: linear in sick history	20.865	5.1
$\beta_{c0}$	Shared leisure: constant	27.512	19.4
$\beta_{c1}$	Shared leisure: linear in age difference	-48.462	-16.0
$\beta_{c2}$	Shared leisure: quadratic term in age difference	21.230	6.5
<b>Stackelberg model (non-cooperative type)</b>			
<i>Wife's utility function</i>			
$a_{f0}$	Household disposable income: constant	8.349	19.1
$a_{f1}$	Household disposable income: linear in wealth	0.064	8.6
$b_f$	Female leisure	29.076	22.1
$b_{fc}$	Shared leisure	2.145	1.8
<i>Husband's utility function</i>			
$a_{m0}$	Household disposable income: constant	0.572	3.7
$a_{m1}$	Household disposable income: linear in wealth	-0.001	-0.1
$b_{m0}$	Male leisure: constant	-1.436	-1.8
$b_{m1}$	Male leisure: linear in sick history	7.118	6.7
$b_{mc}$	Shared leisure	-0.315	-0.6
$\rho$	Correlation	0.446	6.5
<b>Share Parameter</b>			
$\delta$	Proxy for Share of the Stackelberg group in population	-0.477	-9.5
$\lambda_{NC}$	Share of Stackelberg group	0.617	
	Observations	10,008	
	Log-likelihood	-11,841	
	McFadden $R^2$	14.6%	

Most of the estimates have the expected sign. For type C households, the effect of wealth on the marginal utility of disposable income is negative. This is consistent with the common expectation that rich households will value disposable income less, and are thus more likely to take early retirement. The marginal utility of male leisure increases with sick-history, which suggests that males with not-so-good health conditions value their leisure more and they are thus more likely to take early retirement. The parameters associated with the shared leisure term are all sharply determined. In line with the expectation, the marginal utility with respect to shared leisure resembles a parabola. It is positive for most of the relevant age differences, and it decreases as age differences increase up to 12 years. After an age difference of 12 years it begins to climb up very slowly. For type NC households, the estimation results suggest that unobserved variables affecting the utility levels of the spouses are positively correlated. This can be explained by common taste, either due to why they got married in the first place or it had been formed during the long years of adjustments and compromises from both parties. Hiedemann (1998) reports similar results. In contrast to the strong effect of shared leisure found for type C households, it seems that shared leisure has no significant effect on the decision-making in type NC households. For both husband and wife in a non-cooperative household, the coefficient corresponding to shared leisure is not significant from zero. For the husband, similar to the cooperative type, I find a negative wealth effect on marginal utility of disposable income, as well as a positive effect of sick-history on marginal utility of leisure.

## **5.2. Robustness Check on the Functional Form of the Utility Functions**

As discussed earlier, the estimation results presented in Table 4 can be sensitive to the functional form assumptions for the deterministic part of the utility functions (1). This is a common problem in most structural econometric models, due to the lack of theoretical support for choice of functional forms.

One way to solve this problem is to apply the flexible preference model (FPM) of Arthur Van Soest, Marcel Das and Xiaodong Gong (2002). That is, one could use a series expansion to approximate the underlying utility function. This technique avoids the need to specify the utility function explicitly since a polynomial function is capable of approximating any given function to any desired accuracy if the order of the polynomial (denoted hereafter as  $O$ ) can become arbitrarily large. However, three problems hamper its application in empirical practice. First, the choice of  $O$  is arbitrary, only small values of  $O$  can be used, although the consistency requires that  $O$  tends to infinity. Second, quasi-concavity or monotonicity is not guaranteed. John K. Dagsvik and Steinar Strøm (2004) show that FPM can result in quite misleading estimates. Moreover, the computational complexity of the FPM sometimes makes the model estimation impossible. Due to these concerns, I decided not to perform a robustness check based on this framework.

Recently, Dagsvik and Strøm (2004) have provided a justification for the choice of functional forms for the deterministic part of the utility function in a labor supply framework. They show that under certain invariance principles, the utility function is a linear combination of Box-Cox terms.

This suggests a possible robustness test of the functional form assumptions. Estimate the mixed model with the functional forms suggested by Dagsvik and Strøm (2004) then compare the estimates with results in Table 4.

A simplified version of Dagsvik and Strøm's (2004) utility functions suitable for the current setting can be specified as:

$$(17) \quad \begin{aligned} U_m(y_m, y_f) &= a_m \frac{C_{y_m, y_f}^\tau - 1}{\tau} + b_m \frac{L_{m, y_m}^\tau - 1}{\tau} + b_{mc} \frac{L_{y_m, y_f}^\tau - 1}{\tau} + \varepsilon_m(y_m), \\ U_f(y_m, y_f) &= a_f \frac{C_{y_m, y_f}^\tau - 1}{\tau} + b_f \frac{L_{f, y_f}^\tau - 1}{\tau} + b_{fc} \frac{L_{y_m, y_f}^\tau - 1}{\tau} + \varepsilon_f(y_f). \end{aligned}$$

In fact, the specification of utility functions in this paper is a special case of Dagsvik and Strøm's (2004) when  $\tau = 0$ .

I have estimated the model for different values of  $\tau$  ( $\tau \in [0.1, 0.8]$  with steps 0.1<sup>8</sup>). The estimates do not differ very much. The parameter of most interest, the share of type NC households  $\lambda_{NC}$  lies in the interval (0.61, 0.68). A majority of the utility parameters have the same sign and similar numerical magnitude as in Table 4. Based on these findings, I conclude that this paper's results are not very sensitive to the choice of utility functional form.

## 6. Conclusion

Discussions of heterogeneity in traditional models of labor supply are usually focused on two aspects: either differences in preferences or differences in budget constraints. In this paper, I introduce an additional source of heterogeneity for two-person households – unobserved heterogeneity in household decision-making structure. In particular, I assume that there are two latent types of households, where the households of one type behave according to the collective household model while those of the other type behave according to the Stackelberg game model. Consequently, the model has a mixed structure.

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<sup>8</sup> A better way is to take  $\tau$  as an unknown parameter and estimate it together with other parameters in the model and use these estimates as the benchmark. However, the numerical optimization procedure fails to converge in such a setting. It is for the same reason that I am not able to report  $\tau = 0.9$  or 1 in the robustness test.

I apply this framework to analyze the labor market behavior of elderly couples in Norway when a new option (early retirement) becomes available to the husband. The estimation results suggest that more than half of the households in the sample are of the non-cooperative type.

This study demonstrates that the estimation of such a finite mixed model is computationally feasible. I believe that this framework represents a more realistic setting for analyzing household behavior. Although this study focuses on a particular application, the approach developed can readily be applied to other forms of multi-agent choice settings. More importantly, I hope this study will inspire the use of this type of modeling strategy when dealing with unobserved group heterogeneity in other fields of economic research.

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## Stackelberg equilibrium

Given the setting in the paper, a state  $(y_m, y_f)$  is a Stackelberg equilibrium if and only if:

$$(A.1) \quad \begin{cases} U_f(y_m, y_f) > U_f(y_m, 1 - y_f) \\ U_f(1 - y_m, y_f) > U_f(1 - y_m, 1 - y_f) \end{cases} \quad \text{and} \quad U_m(y_m, y_f) > U_m(1 - y_m, y_f)$$

or

$$(A.2) \quad \begin{cases} U_f(y_m, y_f) > U_f(y_m, 1 - y_f) \\ U_f(1 - y_m, y_f) < U_f(1 - y_m, 1 - y_f) \end{cases} \quad \text{and} \quad U_m(y_m, y_f) > U_m(1 - y_m, 1 - y_f)$$

Substitute the utility definition (1) into (A.1) and (A.2), we then have conditions on random variables  $e_m$  and  $e_f$ , which are the differences of the utility error terms and are defined as

$$e_m = \varepsilon_m(1) - \varepsilon_m(0) \quad \text{and} \quad e_f = \varepsilon_f(1) - \varepsilon_f(0) \quad \text{respectively.}$$

Take alternative (1,1) as an example:

It is a Stackelberg equilibrium if and only if:

$$\begin{cases} e_f > v_f(1, 0) - v_f(1, 1) \\ e_f > v_f(0, 0) - v_f(0, 1) \end{cases} \quad \text{and} \quad e_m > v_m(0, 1) - v_m(1, 1)$$

or

$$\begin{cases} e_f > v_f(1, 0) - v_f(1, 1) \\ e_f < v_f(0, 0) - v_f(0, 1) \end{cases} \quad \text{and} \quad e_m > v_m(0, 0) - v_m(1, 1)$$

It follows immediately that the probability of state (1,1) to be a Stackelberg Equilibrium will equal to

$$(A.3) \quad \begin{aligned} \Pr(1, 1) = & \Pr(e_f > \max(v_f(1, 0) - v_f(1, 1), v_f(0, 0) - v_f(0, 1)) \text{ and } e_m > v_m(0, 1) - v_m(1, 1)) \\ & + \Pr(v_f(1, 0) - v_f(1, 1) < e_f < v_f(0, 0) - v_f(0, 1), \text{ and } e_m > v_m(0, 0) - v_m(1, 1)) \end{aligned}$$

It is just simple repetition to calculate the probabilities for other states, and the results is listed in Table A.1.

**Table A.1 Stackelberg equilibrium (SE) (male as the leader)**

Female utility comparison	Female utility error term requirement	Male utility comparison	Male utility error term requirement	SE
$U_f(1,1) - U_f(1,0) > 0$ $U_f(0,1) - U_f(0,0) > 0$	$e_f > \max[v_f(1,0) - v_f(1,1), v_f(0,0) - v_f(0,1)]$	$U_m(1,1) - U_m(0,1) > 0$ $U_m(1,1) - U_m(0,1) < 0$	$e_m > v_m(0,1) - v_m(1,1)$ $e_m < v_m(0,1) - v_m(1,1)$	(1,1) (0,1)
$U_f(1,1) - U_f(1,0) > 0$ $U_f(0,1) - U_f(0,0) < 0$	$v_f(0,0) - v_f(0,1) > e_f > v_f(1,0) - v_f(1,1)$	$U_m(1,1) - U_m(0,0) > 0$ $U_m(1,1) - U_m(0,0) < 0$	$e_m > v_m(0,0) - v_m(1,1)$ $e_m < v_m(0,0) - v_m(1,1)$	(1,1) (0,0)
$U_f(1,1) - U_f(1,0) < 0$ $U_f(0,1) - U_f(0,0) > 0$	$v_f(1,0) - v_f(1,1) > e_f > v_f(1,0) - v_f(1,1)$	$U_m(1,0) - U_m(0,1) > 0$ $U_m(1,0) - U_m(0,1) < 0$	$e_m > v_m(0,1) - v_m(1,0)$ $e_m < v_m(0,1) - v_m(1,0)$	(1,0) (0,1)
$U_f(1,1) - U_f(1,0) < 0$ $U_f(0,1) - U_f(0,0) < 0$	$e_f < \min[v_f(0,0) - v_f(0,1), v_f(1,0) - v_f(1,1)]$	$U_m(1,0) - U_m(0,0) > 0$ $U_m(1,0) - U_m(0,0) < 0$	$e_m > v_m(0,0) - v_m(1,0)$ $e_m < v_m(0,0) - v_m(1,0)$	(1,0) (0,0)

However, before we can specify the likelihood function we need to notice that the probability equation (A.3) involves a max operation, which may cause the likelihood function to be non-differentiable. If the likelihood function is non-differentiable, we will not be able to use gradient-based optimization algorithm such as BFGS, more seriously the MLE estimator will lose its nice asymptotic properties, which make the normal inference inappropriate.

Fortunately, given the empirical setting of the deterministic part of the utility function, the problem does not exist. Once again, take the state (1,1) as an example:

Using the utility function specification (7), we have

$$\begin{aligned}
 & \max(v_f(1,0) - v_f(1,1), v_f(0,0) - v_f(0,1)) \\
 \text{(A.4)} \quad & = a_f \max(\ln(C_{10}/C_{11}), \ln(C_{00}/C_{01})) + b_f \ln(L_{f0}/L_{f1}) + b_{fc} \ln(L_{10}/L_{11}) \\
 & = \begin{cases} a_f \ln(C_{10}/C_{11}) + b_f \ln(L_{f0}/L_{f1}) + b_{fc} \ln(L_{10}/L_{11}) & \text{if } C_{10}/C_{11} > C_{00}/C_{01} \\ a_f \ln(C_{00}/C_{01}) + b_f \ln(L_{f0}/L_{f1}) + b_{fc} \ln(L_{10}/L_{11}) & \text{otherwise} \end{cases}
 \end{aligned}$$

So for any given household, since the relationship between  $C_{10}/C_{11}$  and  $C_{00}/C_{01}$  is fixed, the probability doesn't involve any max operation. Thus the likelihood function doesn't have the non-differentiable problem.

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