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Accounting for differences in choice opportunities in analyses of energy expenditure

Abstract:
Zero expenditure poses several challenges when estimating demand systems. Zero expenditure on energy goods occur due to limited opportunity to consume the good or because the household chooses not to use all available equipment (corner solution). In this paper we develop a method to estimate an Almost Ideal Demand System (AIDS) of household energy demand simultaneously using a Maximum Likelihood approach. The multivariate density of energy expenditures depends on the consumption opportunity of the individual household. We model the choice of corner solutions by a stochastic Kuhn-Tucker condition, and distinguish between zero expenditure due to limited consumption opportunities and corner solutions by using a Double Hurdle model. We find that accounting for zero expenditure in the estimation has a significant effect on the estimated parameters. Assuming stochastic interdependence between expenditures on different energy goods within the household, in addition to accounting for zero expenditure, has only a minor effect on the estimated coefficients.

Keywords: Residential energy consumption, AIDS model with full price index, zero expenditure, stochastic Kuhn-Tucker condition, double hurdle model, multivariate distribution.

JEL classification: D12, Q41

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1. Introduction

When analyzing household demand for specific goods using micro data, we frequently observe households with zero expenditure on some goods. Household energy consumption serves as a typical example of a consumption group with widespread occurrence of zero expenditure. Zero expenditure on energy goods may either occur because the household does not own heating equipment giving opportunity to use the good (limited consumption opportunities), or because the household does not utilize all available equipment (corner solution).

Most studies accounting for zero expenditure use econometric models and focus on the demand for a single good (see e.g. Fry and Pashardes, 1994, Yen and Jones, 1997, Burton et al., 1994, Mihalopoulos and Demoussis, 2001, Newman et al., 2003, Deaton and Irish, 1984, Garcia and Labeaga, 1996, Atkinson et al., 1990). However, some attempts have been made to estimate an entire demand system accounting for zero expenditure. One of the most comprehensive studies accounting for zero expenditure in a demand system is Wales and Woodland (1983). They specify two methods of modeling zero expenditure; one stochastic Kuhn-Tucker approach and one Amemiya-Tobin type of modeling zero expenditure econometrically. Both methods are applied to a Maximum Likelihood (ML) estimation of a Linear Expenditure System (LES). Results from the two approaches are compared and discussed. They find that the results are not very sensitive to the method used. However, they prefer the Kuhn-Tucker specification based on theoretical grounds.

After Wales and Woodland's article, very few attempts have been made estimating a multivariate expenditure system where zero expenditure is modeled explicitly. Later studies follow one of two alternative approaches: One approach is to apply an econometric model and the other approach is to use the duality properties of the cost and utility functions to model zero expenditures. Heien and Wessells (1990) use a two-stage Amemiya (1974) approach to estimate an Almost Ideal Demand System (AIDS) for eleven food items.[1] Cronick et al. (1994) estimate Engle curves using a multivariate Tobit model to account for zero expenditure assuming interdependence between the goods. In a recent study, Golan et al. (2001) develop an estimation procedure based on Generalized Maximum Entropy (GME) theory to estimate an AIDS system with no-negativity constraints. Since Golan et al. focus on the estimation method, the economic modeling of zero expenditure is not discussed. Fry et al. (2001) is the only recent study we found modeling zero expenditure using the

[1] As noted by Ransom (1987), the two-stage Amemiya approach and the simultaneous approach in Wales and Woodland (1983) coincide in the case of constant prices, and the simpler two-stage Amemiya approach may be applied.

duality properties. They apply a compositional data analysis on a modified AIDS model to estimate the demand for, among other things, cigarettes and tobacco. In this study, they do not distinguish between zero expenditure for consumers who smoke and consumers who do not. Abstention is often modeled explicitly in the single equation studies of tobacco and cigarette consumption applying e.g. a Double Hurdle (DH) model (see e.g. Garcia and Labeaga, 1996).

In this study, we combine several approaches found in the literature. We apply an AIDS model like Heien and Wessells (1990), Golan et al. (2001) and Fry et al. (2001) to describe household expenditures. All these studies apply a Stone index to approximate the price index within the AIDS model. In our analysis, we include the full price index directly in the estimation. The reason is that using the Stone index may result in biased estimates, since it includes variables that are endogenous to the consumer (see e.g. Pashardes, 1993 for a discussion). To distinguish between zero expenditure on energy goods due to limited consumption opportunities or corner solutions, we apply a DH model like in e.g. Garcia and Labeaga (1996). To our knowledge, using a DH model estimating an expenditure system (and not a single good) has never previously been documented in the literature. Furthermore, we model the choice of corner solutions in terms of a stochastic Kuhn-Tucker optimization problem similar to the specification in Wales and Woodland (1983). Except for the Wales and Woodland study, we have not been able to find any studies describing the Kuhn-Tucker conditions for zero expenditure estimating a demand system. We modify the DH model to fit the Kuhn-Tucker conditions, in order to give the probabilities of observing corner solutions in the DH model an economic interpretation. Finally, we reframe the AIDS model to account for differences in consumption opportunities across households. In the literature, we have found no studies discussing differences in the individual households' opportunities to consume various goods and how this will affect the demand system. Wales and Woodland (1983) specify different expenditure systems for households who choose to consume different combinations of goods. In our study, we adjust the likelihood as well as the AIDS model to account for the consumption opportunities given by the individual household's heating portfolio.

The paper is organized as follows: In Section 2, we describe the data. Section 3 presents the theoretical framework for household energy expenditure and discusses how differences in consumption opportunities affect the properties of the model. In Section 4 we discuss how to modify a double hurdle model to adjust the estimations for zero expenditure and reflect the Kuhn-Tucker conditions for choosing a corner solution. We also discuss how to specify the likelihood function of the multivariate double hurdle model with interdependence between expenditures on different goods within each household. Section 5 presents the empirical results, and finally in Section 6, some concluding remarks are made.

2. The data

The data used in this analysis originate from different sources and contain information on 3,434 individual households observed in 1993, 1994 and 1995. The main data source is the annual Norwegian Survey of Consumer Expenditure (SCE) (see Statistics Norway, 1996), where the households are interviewed in person. Our data include information about consumption of and expenditure on paraffin, fuel oil and firewood, as well as electricity expenditure for the last 12 months prior to the interview. Thus, problems with rare purchases are not as severe here as in analyses based on data from expenditure surveys where only purchases for the last 14 days are recorded (see e.g. Deaton and Irish, 1984). However, firewood, paraffin and fuel oil may be stored for longer periods than one year. Thus, all estimates should be interpreted as effects on the acquirement of firewood, paraffin and fuel oil, and not consumption. In the analysis, we look at the combined expenditures on both paraffin and fuel oil (hereafter referred to as fuel oils). The household may have zero expenditure on firewood even with a positive consumption and no storage, e.g. if the wood is acquired by own labor or as a gift. This is quite common in Norway, where many households use considerable time and effort cutting and chopping firewood (hereafter referred to as free firewood). The SCE contains information about the amount of free firewood the household has acquired during the last 12 months prior to the interview. The SCE also contains information about electricity supplier, ownership of durables, heating technology and household and dwelling characteristics.

Information on prices and temperature is collected from different sources. Oil and firewood prices are obtained from the SCE, calculated as the expenditure divided by physical amount for households reporting both a positive expenditure on and a physical amount of the good. These prices are averaged by county and applied for households in the respective county who do not have both a positive expenditure on and amount of firewood and/or paraffin and fuel oil during the last 12 months. Furthermore, households who have a positive acquired quantity of free firewood are assumed to have an alternative cost on this wood equal to the mean price of purchased firewood in each county.[2] This is done to include free firewood in the estimation of the properties of the demand functions. The shares of purchased and free firewood are used as weights to obtain the mean firewood price for households with both a positive expenditure on firewood and a positive acquisition of free firewood. We use this

[2] We use the market price as the alternative value/cost of free firewood. First, the alternative to cutting wood is to purchase it at the market price. Second, the household may sell free firewood instead of consuming it themselves.

information to calculate the total expenditures on firewood, including the alternative value of free firewood. Information on electricity prices is collected from the households' individual electricity suppliers and the Norwegian Water Resources and Energy Directorate. If the price information for a household is missing, the mean price of all power suppliers distributing to the household's area of residence (municipality) is allocated to the individual household.[3] The Norwegian Institute of Meteorology provides annual information about regional variations in temperature for all municipalities included in the SCE.

In Table 1, we report mean values for key variables by heating portfolio. We divide the sample into four mutually exclusive groups depending on their opportunity to consume various energy goods: 1) households that are only able to use electricity for space heating, 2) households that may use fuel oil (including paraffin) as well as electricity, but not firewood, 3) households that may use firewood in addition to electricity, but not fuel oil and 4) households that may use both firewood and fuel oil in addition to electricity.

Table 1. Mean values by heating portfolio

	1) Electricity only	*2) Electricity and fuel oils*	*3) Electricity and firewood*	*4) Electricity, fuel oils and firewood*
Sample share (%)	15.5	4.1	56.5	23.9
Total energy budget (NOK)	7 182	11 856	12 664	13 456
Total energy consumption (kWh)	16 381	27 107	28 509	30 488
Electricity consumption (kWh)	16 381	19 758	25 069	23 995
Consumption of fuel oils (kWh)		7 348		4 526
Total firewood consumption (kWh)			3 439	1 967
Net floor space (m^2)	90	113	137	137
Share with positive fuel oil expenditure (%)		77		55
Share with positive firewood expenditure (%)			74	57

Table 1 shows that the most common heating portfolio in our sample is based on electricity and firewood (group 3), and the least common heating portfolio is based on electricity and fuel oils only (group 2). Approximately 80 percent of the households have the opportunity to use firewood (group 3 and 4), and 28 percent of the sample may use fuel oils (group 2 and 4). Furthermore, approximately 16 percent of the sample can only use electricity for heating (group 1) and have no opportunity in the short run to switch between energy carriers when relative prices change.

[3] In this period, most households used their local power distributor.

All groups use electricity as their main source of energy. Households in group 4 have the highest total energy consumption, whereas households in group 1, that may only use electricity, have the lowest mean value. The main reason why households in group 1 have the lowest total energy consumption is that these households have smaller residences than the other groups. The largest net floor space is found in groups 3 and 4, which include households with the ability to use firewood for space heating. Looking at the firewood consumption, the mean value for households that may only use firewood in addition to electricity (group 3) is higher than the mean value for household that may also use fuel oils (group 4). Finally, we see that not all households utilize their consumption opportunities. Only 57 percent of the households in group 4 and 74 percent in group 3 acquired firewood. The shares of households with positive expenditure on fuel oils in groups 2 and 4 are 77 and 55 percent, respectively. In the sample, only 58 percent of households with the opportunity to consume fuel oils have a positive expenditure. The percentage for firewood is 69. That is, 42 and 31 percent of households with the opportunity to consume fuel oils and firewood respectively, choose a corner solution.

3. Modeling household expenditure and corner solutions

Norwegian households have different heating technologies and thus different opportunities to consume various energy goods. Furthermore, not all households choose to make use of all the heating equipment available, because consuming other goods may give higher utility at a given set of prices. Thus, we need to model the households' decision to use or not use available heating equipment, and modify the model for differences in consumption opportunities across households. To model the choice of corner solutions given the opportunity to consume a particular good, we apply a stochastic Kuhn-Tucker maximization problem, similar to the one described in Wales and Woodland (1983). Here, we assume the households' choice of heating technology is not affected by changes in prices. That is, we study short-term effects of changes in prices on the utilization of the already existing heating portfolio.

3.1 The Kuhn-Tucker problem

We assume that household *h* derives utility (*U*) from the consumption of a vector of goods $\left(q^h\right)$, including energy goods $\left(i=1,2,3\right)$, which are available to the household at a set of prices p (vector). Furthermore, we assume that each household only considers buying goods it has the opportunity to consume. The household is assumed to maximize its utility with respect to the consumption of all available goods subject to its budget, that is its total expenditure $\left(x^h = p\,q^h\right)$ is less or equal to household income $\left(m^h\right): x^h = p\,q^h \leq m^h$, and the household cannot have a negative consumption of any good; $q^h \geq 0$. This gives the following optimization problem:

$$(1) \qquad \max_{q^h} U^h\left(q^h\right): p\,q^h \leq m^h, \quad q^h \geq 0 .$$

The utility function is assumed to be continuously differentiable, quasi-concave and increasing in the consumption of all goods. Since the utility function is increasing in the consumption of all goods, the consumer will spend all his income, and at least one good will be consumed. Without any loss of generality, we assume that this is the fourth good, *i = 4*, which contains all consumption excess energy goods. Necessary and sufficient condition for utility maximization given this optimization problem may be written as (see e.g. Wales and Woodland, 1983, for a discussion):

$$(2) \qquad \begin{aligned} &\frac{p_4 U_i'}{m^h} - \frac{p_i U_4'}{m^h} \;\leq\; 0 \;\leq\; q_i^h \\ &p\,q^h = m^h \end{aligned}$$

The consumption of good *i* equals zero $\left(q_i^h \;=\; 0\right)$ if the marginal rate of substitution is less than the price ratio $\left(\frac{U_i'}{U_4'} \;<\; \frac{p_i}{p_4}\right)$ for all units of consumption. Otherwise, the household has a positive consumption of good *i* ($q_i^h > 0$), and the optimal consumption is characterized by equality between the marginal rate of substitution between goods *i* and 4 and the price ratio $\left(\frac{U_i'}{U_4'} \;=\; \frac{p_i}{p_4}\right)$. That is, the household will only have a positive consumption of energy good *i* if the marginal utility of consuming the first unit relative to the marginal utility of increasing other consumption exceeds the relative cost of this consumption. If not, it will choose a corner solution and not consume the good. This optimization problem leads to the demand functions for all goods the household has the opportunity to consume as a function of all prices and income:

$$(3) \qquad q_i^h = q_i^h\left(p, m^h\right) \;\geq\; 0 .$$

3.2 Econometric specification

From the individual household's point of view the utility function is assumed to be non-stochastic. Thus, the household knows if it wants to consume a good and, if so, its expenditures on the good at different sets of prices and income. However, from the researcher's point of view, both utility and expenditures are stochastic, as we assume that differences in individual tastes are randomly distributed over the population. Thus, we need to specify the probability of observing zero expenditure, and the functional form and stochastic specification of the expenditure function.

In our estimation, we focus on the expenditure on energy goods. The expenditures on other goods $(i = 4)$ are viewed as residual and thus not discussed further in this paper. We study three energy goods: Electricity $(i = 1)$, fuel oils (paraffin and fuel oil) $(i = 2)$ and firewood $(i = 3)$.

3.2.2 Corner solutions

We follow the approach in Wales and Woodland (1983) and assume that the marginal utility consists of a common deterministic ($\underline{U_i'}$) and a random (ω_i^h) component: $U_i' = \underline{U_i'} + \varpi_i^h$.

Using the Kuhn-Tucker conditions for optimization, we may express the probability of observing a zero expenditure on good *i* as a function of whether the marginal rate of substitution is less than the price ratio, given by:

(4) $$P\left(q_i^h = 0\right) = P\left(p_4 U_i' - p_i U_4' < 0\right) = P\left(p_4 \varpi_i^h - p_i \varpi_4^h \le p_i \underline{U_4'} - p_4 \underline{U_i'}\right) = P\left(\underline{\psi_i^h}\right)$$

where $\underline{\psi_i^h}$ indicates how the difference $p_i \underline{U_4'} - p_4 \underline{U_i'}$ is affected by differences in relative prices in optimum. $\psi_i^h = \underline{\psi_i^h} + \xi_i^h$ is approximated by a linear function, given by:

(5) $$\psi_i^h = \psi_{i0} + \psi_{i1} p_1 + \psi_{i2} p_2 OE_2^h + \psi_{i3} p_3 OE_3^h + \xi_i .$$

where $\xi_i^h = p_4 \varpi_i^h - p_i \varpi_4^h$ is the sum of all the stochastic components. We use a dummy variable, OE_j^h, to indicate whether the household has opportunities to consume good *j* (OE_j^h=1) or not (OE_j^h=0). This is done to exclude prices the household does not have the opportunity to consume. The stochastic component, ξ_i^h, is assumed to be independent and identically normally distributed with a zero expectation and a constant variance. The probability of observing a positive expenditure on good *i* is given by $P\left(q_i^h > 0\right) = 1 - P\left(q_i^h = 0\right)$.

3.2.2 The expenditure system

Given a positive expenditure on good *i*, we assume that an Almost Ideal Demand System (AIDS) describes the distribution of the total budget on different goods.[4] Household *h*s budget share on good *i* (w_i^h), given a positive expenditure on this good, is given by:

$$(6) \qquad w_i^h = \alpha_i^h + \sum_j^{J_h} \gamma_{ij}^h \log\left(p_j^h\right) + \beta_i^h \left[\log\left(x^h\right) - \log\left(P^h\right)\right],$$

where $\log P^h = \alpha_0 + \sum_{k=1}^{J_h} \alpha_k^h \log\left(p_k^h\right) + \tfrac{1}{2} \sum_{k=1}^{J_h} \sum_{j=1}^{J_h} \gamma_{jk}^h \log\left(p_k^h\right) \log\left(p_j^h\right)$, P^h is a price index of goods each household has the opportunity to consume, x^h is the total budget of household *h* and p_j^h is the price of good *j* for household *h*. Note that we sum over all $k = 1, .., J_h$ and $j = 1, .., J_h$, where J_h is a vector of goods household *h* has the opportunity to consume.

Since the utility function is randomly distributed over all households in the population, expenditures are also randomly and normally distributed. For households with positive expenditure on good *i* (y_i^h), the expenditure is assumed to be the sum of a deterministic component measuring the expected expenditure on good *i* (μ_i^h) and a stochastic component (ε_i^h), given by:

$$(7) \qquad \begin{aligned} y_i^h &= \mu_i^h + \varepsilon_i^h = \left[\alpha_i + \sum_j^J \gamma_{ij} \log\left(p_j^h\right) OE_j^h + \beta_i \left(\log\left(x^h\right) - \log\left(P^h\right)\right)\right] x^h + \varepsilon_i^h , \\ \log P^h &= \alpha_0 + \sum_{k=1}^K \alpha_k \log\left(p_k^h\right) OE_k^h + \tfrac{1}{2} \sum_{k=1}^K \sum_{j=1}^J \gamma_{jk} \log\left(p_k^h\right) OE_k^h \log\left(p_j^h\right) OE_j^h . \end{aligned}$$

Equation (7) is a stochastic specification of the expenditure on good *i*, that is, the budget share in equation (6) multiplied by the total budget. In equation (6), only prices of goods the household has the opportunity to consume enter the expenditure function. To ensure that only these prices are included in equation (7), we multiply the logarithms of all prices by a dummy (OE_j^h), which indicates whether the

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[4] See e.g. Deaton and Muellbauer (1980) for more information about the AIDS model.

household has opportunity to consume good *j*. Since the dummy equals zero for households who cannot consume a particular good, all prices of goods the household cannot consume are excluded.

We assume that the stochastic term, ε_i^h, is independently and identically distributed with zero expectation, $E\left(\varepsilon_i^h\right)=0$, and a constant variance across households, $E\left(\varepsilon_i^h \varepsilon_i^r\right)=\sigma_i$. We also assume that the stochastic terms are correlated between goods within a household, $E\left(\varepsilon_i^h \varepsilon_j^h\right)=\sigma_{ij}^h$, but uncorrelated across households, $E\left(\varepsilon_i^h \varepsilon_j^r\right)=0$.

4. The likelihood function

As seen in Table 1, the occurrence of zero expenditure for fuel oils and firewood is quite substantial. There are two main reasons for observing zero expenditure; either limited consumption opportunities or corner solutions. We apply a Double Hurdle (DH) model to separate households without the opportunity to consume from households with corner solutions. In Section 4.1 we describe the standard DH model and our modifications of the model to fit the Kuhn-Tucker conditions.

The DH model described in Section 4.1 is specified for a single equation. Since our aim is to estimate the expenditure system, we need to specify how the simultaneous multivariate density is affected by the occurrence of zero expenditure. Due to common effects on consumption of energy goods within a household (income, environmental friendly attitude, number of household members, net floor space, etc.), the consumption of different energy goods is assumed to be stochastically interdependent. In order to account for zero expenditure on some (but not all) goods in the simultaneous multivariate likelihood function, we need to decompose it into its conditional counterparts. This decomposition is discussed in Section 4.2, and in Section 4.3, we combine all the elements to specify the simultaneous likelihood function to be estimated.

4.1 The double hurdle model

In a double hurdle model, the probability density is a discrete-continuous mixture of households with a positive expenditure and households with zero expenditure on the good:

$$(8) \qquad f\left(y_j^h\right)=\begin{bmatrix} f_+\left(y_j^h\right) & if \quad y_j^h > 0 \\ f_0 & if \quad y_j^h = 0, \end{bmatrix}$$

where the discrete component, f_0, is the probability mass measured at zero expenditure and the continuous component, $f_+\left(y_j^h\right)$, is the density for households with a positive expenditure.[5]

Households who have zero expenditure because they do not have the opportunity to consume good *j* are characterized by zero value of the dummy variable OE_j^h ($OE_j^h = 0$). Households choosing a corner solution are characterized by zero expenditure conditional on the opportunity to consume ($y_j^h = 0 \mid OE_j^h = 1$). A household may only have a positive expenditure if it has the opportunity to consume. This gives a probability of a positive expenditure of $P\left(y_j^h > 0, OE_j^h = 1\right) = P\left(OE_j^h = 1\right)P\left(y_j^h > 0 \mid OE_j^h = 1\right)$. The probability of zero expenditure (f_0) is $1 - P\left(OE_j^h = 1\right)P\left(y_j^h > 0 \mid OE_j^h = 1\right)$.

In our model, we focus on the consumption of energy goods in the short run for a given choice of heating technology. Since the choice of equipment ownership is assumed to be exogenous, the choice of corner solutions is stochastically independent on equipment ownership, and thus the probability of having a positive expenditure conditional on the possibility of consuming a particular good equals the marginal probability: $P\left(y_j^h > 0 \mid OE_j^h = 1\right) = P\left(y_j^h > 0\right)$. This means that we, in this analysis, apply a double hurdle model with independence (see e.g. Garcia and Labeaga, 1996, for a discussion). Given a short-run analysis where the stock of heating equipment is given from previous decisions, the continuous part of the distribution is given by: $f_+\left(y_j^h\right) = f\left(y_j^h \mid y_j^h > 0\right)P\left(y_j^h > 0\right)P\left(OE_j^h = 1\right)$, where $f\left(y_j^h \mid y_j^h > 0\right)$ is the truncated density function.

Assuming expenditures to be independently and identically distributed, the likelihood function in the DH model is the product of all densities for all households, that is:

$$(9) \quad L = \prod_{h_+} f_+\left(y_j^h\right) \prod_{h_0} f_0 = \prod_{h_+} f\left(y_j^h \mid y_j^h > 0\right)P\left(y_j^h > 0\right)P\left(OE_j^h = 1\right) \prod_{h_0} 1 - P\left(OE_j^h = 1\right)P\left(y_j^h > 0\right)$$

This equals the Cragg specification of the DH model if the distributions are assumed to be normal (see Smith, 2002 or Cragg, 1971). Equation (9) represents the likelihood function for a single equation DH model with independence.

[5] See e.g. Smith (2002) or Garcia and Labeaga (1996) for a description of the double hurdle model.

4.1.1 Modifications of the single equation double hurdle model

As noted by Smith (2002), it is assumed in the DH model that it is not possible to separate different sources of zero observations in the data. In our data, however, we are able to identify whether the household has zero expenditure due to limited consumption opportunities or due to corner solutions. Thus, we want to decompose the discrete part of the DH model to represent these two types of zero expenditure. To du this, we apply a property of the discrete prart of the distribution discussed in Smith (2002), equation (6): $f_0 = P(OE_j^h = 0) + P(y_j^h = 0) - P(OE_j^h = 0)P(y_j^h = 0)$. Rearranging this, we may decompose the discrete part of the density into the probability of not having the opportunity to consume good *j*, $P(OE_j^h = 0)$, and the probability of having the opportunity to consume but choosing a corner solution, $P(y_j^h = 0)[1 - P(OE_j^h = 0)]$. This property is inserted into the likelihood.

Furthermore, we combine the DH model with the Kuhn-Tucker problem described in Section 3.2.2 to specify the probability of observing corner solutions in terms of whether the marginal rate of substitution is less than the price ratio: $P(y_j^h = 0) = P\left(\frac{U'_j}{U'_4} < \frac{p_j}{p_4}\right)$. Using the stochastic specification, we have $P(y_j^h = 0) = P\left(p_4 \varpi_j^h - p_j \varpi_4^h \le \underline{p_j U'_4} - \underline{p_4 U'_j}\right) = P\left(\underline{\psi_j^h}\right)$. Inserting this into equation (9), and the decomposition of the discrete part of the density, we obtain the following likelihood function for our modification of the single equation DH model.

(10) $$L = \prod_{\substack{OE_j^h = 1 \\ y_j^h > 0}} f\left(y_j^h \mid y_j^h > 0\right)\left[1 - P\left(\underline{\psi_j^h}\right)\right] P\left(OE_j^h = 1\right) \prod_{\substack{OE_j^h = 1 \\ y_j^h = 0}} P\left(\underline{\psi_j^h}\right) P\left(OE_j^h = 1\right) \prod_{OE_j^h = 0} P\left(OE_j^h = 0\right)$$

In the continuous part of the density (f_+), the probability of having a positive expenditure is expressed in terms of the probability of opportunity of consuming energy good *j*, the probability of not having a marginal rate of substitution less than the price ratio and the density function for the expenditure of good *j* (the first part of the likelihood function). The discrete component of the density (f_0) is divided into two components; the probability of not having the opportunity to consume good *i* (the last part of the likelihood function), and the probability of choosing a corner solution given the opportunity to consume (the second part of the likelihood function).

4.2 Decomposing the multivariate distribution

The likelihood function for our modification of the DH model in equation (10) is specified for a single good. If the expenditures on different energy goods are stochastically independent, we may estimate the likelihood functions for each household separately. However, we have reasons to believe that the consumption of different energy goods is interdependent within a household. This is both due to common budget effects, omitted characteristics of the individual household with effect on energy consumption and the capacity of different heating equipment: Stoves for fuel oil, paraffin and firewood have high effect even at minimum intensity. When these stoves are put into use, the use of electricity may be reduced if the increase in power requirement is more than satisfied by the capacity of these stoves.

In order to allow for zero expenditure in the firewood and fuel oil functions, we decompose the simultaneous multivariate density for the expenditure on all energy goods, $f\left(y_1^h, y_2^h, y_3^h\right)$, into its conditional counterparts: $f\left(y_1^h, y_2^h, y_3^h\right) = f\left(y_1^h \mid y_2^h, y_3^h\right) f\left(y_3^h \mid y_2^h\right) f\left(y_2^h\right)$. We assume that the electricity expenditure is conditional on the expenditure on firewood and fuel oils and that the firewood expenditure is conditional on expenditure on fuel oils. This way of conditioning fitted our data best.

The simultaneous likelihood, assuming that all households have the opportunity to consume all energy goods, is given by:

(11) $$L = \prod_{h=1}^{H} f\left(y_1^h, y_2^h, y_3^h\right) = \prod_{h=1}^{H} f\left(y_1^h \mid y_2^h, y_3^h\right) f\left(y_3^h \mid y_2^h\right) f\left(y_2^h\right)$$

However, the opportunity to consume various energy goods differs across households. If a household can use both fuel oils and firewood in addition to electricity for space heating, the household is assigned a trivariate distribution. If it may use one alternative to electricity for space heating, it is assigned a bivariate distribution. If it may use electricity only, it is assigned a univariate distribution. We use the dummy variables OE_j^h, indicating equipment ownership, to make the distribution of electricity and firewood expenditure dependent on the expenditure on alternative energy goods depending on the household's consumption opportunities. How this is done is described in further detail in the next section.

4.3 The multivariate likelihood function corrected for zero observations

Finally, we combine our specification of the DH model (described in equation 11, Section 4.1) and the decomposition of the simultaneous multivariate density function (described in equation 12, Section 4.2) in a simultaneous multivariate DH model where we account for different consumption possibilities across households. Assuming the expenditures on all available energy goods to be simultaneously normally distributed, the simultaneous multivariate likelihood function is given by:

(12)

$$
\begin{aligned}
L = & \prod_h \frac{1}{\sigma_{1.23}^h} \phi\left(\frac{y_1^h - \mu_{1.23}^h}{\sigma_{1.23}^h}\right) \\
& \prod_h \left\{\kappa_3 \frac{1}{\sigma_{3.2}^h} \phi\left(\frac{y_3^h - \mu_{3.2}^h}{\sigma_{3.2}^h}\right)\left(1 - \Phi\left(\underline{\psi}_3^h\right)\right)\right\}^{D_3^h OE_3^h} \prod_h \left\{\Phi\left(\underline{\psi}_3^h\right)\kappa_3\right\}^{\left(1-D_3^h\right)OE_3^h} \prod_h \left(1-\kappa_3\right)^{\left(1-OE_3^h\right)} \\
& \prod_h \left\{\kappa_2 \frac{1}{\sigma_2} \phi\left(\frac{y_2^h - \mu_2^h}{\sigma_2}\right)\left(1 - \Phi\left(\underline{\psi}_2^h\right)\right)\right\}^{D_2^h OE_2^h} \prod_h \left\{\Phi\left(\underline{\psi}_2^h\right)\kappa_2\right\}^{\left(1-D_2^h\right)OE_2^h} \prod_h \left(1-\kappa_2\right)^{\left(1-OE_2^h\right)}
\end{aligned}
$$

where κ_i denotes the share of households not having the opportunity to consume good *i*, indicating the probability of belonging to this household group. φ denotes the normal density function and Φ denotes the normal probability function. Thus, $\Phi\left(\underline{\psi}_j^h\right)$ denotes the probability of choosing a corner solution for energy good *j* given the opportunity to consume good *j*. $\mu_{1.23}^h$ and $\mu_{3.2}^h$ are the expected means of the electricity and firewood expenditures conditional on alternative fuels available to the household, and μ_2^h is the expected mean of fuel oils. $\sigma_{1.23}^h$ and $\sigma_{3.2}^h$ are the standard deviations of the electricity and firewood expenditures conditional on alternative fuels available to the household and σ_2^h is the standard deviation of the expenditure on fuel oils. The properties of the conditional density functions depend on the individual household's consumption opportunities. We use the dummy OE_j^h, indicating equipment ownership, to separate households with different consumption opportunities and the dummy D_i^h, indicating if the household has a positive expenditure on energy good *i*, to identify corner solutions.

4.3.1 Adjusting for zero expenditure

To create a DH model, we use the dummy variables OE_i^h and D_i^h to exalt the expressions for the fuel oils and firewood expenditure by a combination of the two (see equation 12). If the household does not have the opportunity to use energy good *i*, OE_i^h=0 and thus $OE_i^h D_i^h$ = 0. This makes the expression for the continuous part of the density in the likelihood function and the discrete part for households choosing a corner solution, equal to 1 (see the first and second part of the second and third line in equation 12). Thus, households without the opportunity to consume energy good *i* are assigned the share of households who are not able to consume energy good *i* in the discrete part of the distribution (see the last part of the second and third line in equation 12). Households who choose a corner solution is assigned the discrete part of the distribution with the opportunity to consume, as OE_i^h = 1 and D_i^h = 0 (see the second part of the second and third line in equation 13). Finally, households with a positive expenditure are assigned to the continuous part of the distribution, since OE_i^h = 1 and D_i^h = 1 for these households (see the first part of the second and third line in equation 12). This ensures that each household is assigned components in the likelihood function according to consumption opportunities and expenditure choice for all energy goods.

4.3.2 Properties of the multivariate density function for different heating portfolios

To ensure that the households get a tri-, bi- or univariate distribution of energy expenditures according to each household's opportunity to consume various energy goods, we use the dummy indicating the consumption opportunities to assign the right properties of the conditional density functions.

The conditional mean and standard deviation for the electricity expenditure in the likelihood function (12) are given by:

(13)

$$\mu_{1.23}^h = \mu_1 + \rho_{12}\sigma_1\left(\frac{y_2^h - \mu_2}{\sigma_2}\right)OE_2^h\left(1-OE_3^h\right) + \rho_{13}\sigma_1\left(\frac{y_3^h - \mu_3}{\sigma_3}\right)OE_3^h\left(1-OE_2^h\right)$$
$$+\left[\frac{\sigma_1}{\sigma_2}\left(\frac{\rho_{12}-\rho_{13}\rho_{23}}{1-\rho_{23}^2}\right)\left(y_2^h-\mu_2\right)+\frac{\sigma_1}{\sigma_3}\left(\frac{\rho_{13}-\rho_{12}\rho_{23}}{1-\rho_{23}^2}\right)\left(y_3^h-\mu_3\right)\right]OE_2^h\,OE_3^h$$

$$\sigma_{1.23}^h = \sqrt{\sigma_1^2\left(1-\rho_{12}^2 OE_2^h\left(1-OE_3^h\right)-\rho_{13}^2 OE_3^h\left(1-OE_2^h\right)-R_{1.23}^2\,OE_2^h\,OE_3^h\right)},$$

where $R_{1.23}^2 = \rho_{13}^2 + \left(\frac{\rho_{12}-\rho_{13}\rho_{23}}{\sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23}^2}}\right)^2 - \rho_{13}^2\left(\frac{\rho_{12}-\rho_{13}\rho_{23}}{\sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23}^2}}\right)^2$.

If the household only may use electricity for space heating, both dummies indicating alternative equipment ownership (OE_j^h, j = 2, 3) equal zero. In this case, the conditional mean and standard deviation in the distribution of electricity expenditure equal the properties in the marginal distribution. If the household may use firewood as an alternative to electricity for space heating, the dummy for firewood equipment ownership equals one and the dummy for fuel oil equipment ownership equals zero. In this case the electricity expenditure is assumed to be conditional on the expenditure on firewood only, and the properties of the density function are those of a conditional bivariate normal density. The case of fuel oil equipment is analogous. If the household may use both fuel oils and firewood as a supplement to electricity for space heating, the properties of the density are those of a conditional multivariate normal density, i.e. the trivariate normal density (see Anderson, 1984, chapter 2.5 for more information). These conditional distributions describe the dependency of the electricity expenditure with respect to the expenditures on fuel oils and firewood.

The case is analogous for the firewood expenditure conditional on the expenditure on fuel oils, where the conditional mean and standard deviation for the firewood expenditure are given by:

(14)
$$\mu_{3.2}^h = \mu_3 + \rho_{32}\sigma_3\left(\frac{y_2^h - \mu_2}{\sigma_2}\right)OE_2^h$$
$$\sigma_{3.2}^h = \sqrt{\sigma_3^2\left(1 - \rho_{32}^2 OE_2^h\right)}$$

5 Comparing results from different model modifications

The estimation of the likelihood function is conducted applying the MINIMIZE procedure in Limdep (Greene, 1995), specifying the simultaneous log-likelihood function in equations (12)-(14) where the AIDS model for household energy expenditure is given by equation (7). In this section we present the results from the estimations.

We wanted to evaluate the effects of adjusting the estimation for zero expenditure. We also wanted to test if the expenditure on one energy good is stochastically independent of the expenditures on other energy goods. The reason for testing stochastic interdependence in household energy consumption is that, if the expenditures are stochastically independent, we may estimate the expenditures on each good separately using the likelihood function for the single equation DH model in (10). This will simplify the estimations considerably.

We start by comparing the results from an AIDS model with no modifications with a model where we adjust the estimation for zero expenditure, but where the expenditures on energy goods are assumed to be stochastically independent within each household, that is $E\left(\varepsilon_i^h \varepsilon_j^h\right) = 0$. Then we compare the model adjusted for zero expenditure with a model adjusted for zero expenditure where we also assume stochastic interdependence, that is we assume $E\left(\varepsilon_i^h \varepsilon_j^h\right) = \sigma_{ij}^h$. All versions of the model are estimated simultaneously.

5.1 Adjusting for zero expenditure

In Table 2 we compare the estimated parameters from the estimation of an AIDS model with no modifications (column 1 of the table) with a model adjusted for zero expenditure only (see column 2). In the last column, we report the difference in the estimated coefficients in the two models, and indicate the level of significance of this difference. Section A of the table gives the results for the electricity expenditure, section B gives the results for the fuel oil and paraffin expenditure and section C gives the results for the firewood expenditure. In section B and C we report the estimated parameters of the expenditure function for households with a positive expenditure and the estimated parameters in the probability of choosing a corner solution.

We see from Table 2 that adjusting the estimation for zero expenditure only affects two coefficients in the electricity expenditure in a significant way, and most coefficients maintain their level of significance. Looking at the fuel oils and firewood expenditure, the impact of adjusting the estimation for zero expenditure is much more evident as most coefficients change significantly. The exception is γ_{31}, which does not differ significantly from zero in any of the models. Furthermore, the estimates of γ_{13} and γ_{33} change sign and become insignificant. Thus, adjusting for zero expenditure is important when estimating expenditure on firewood and fuel oils. The reason is the large number of zero observations on fuel oils and firewood. Only 28 percent of households have the opportunity to consume fuel oils ($\hat{\kappa}_2 = 0.280$) and 80 percent of households have the opportunity to consume firewood ($\hat{\kappa}_3 = 0.804$). Of these households, only 58 and 69 percent had a positive expenditure on fuel oils and firewood respectively.

Looking at the probability of choosing a corner solution, we see that when the electricity price increases, the probability of observing corner solutions of both fuel oils and firewood is reduced since both the sign of ψ_{31} and ψ_{21} are negative. That is, when the price of electricity increases, households use their stock of alternative heating equipment more than before. This effect is, however, only significant for the probability of choosing a corner solution for firewood. We also see that an increase in the firewood price increases the probability of observing a corner solution for firewood (ψ_{33}). However, the effect of fuel oil price on the probability of observing zero fuel oil consumption is significantly negative (ψ_{22}). Furthermore, the cross price effects between fuel oils and firewood (ψ_{23}, ψ_{32}) are positive, i.e. the cross price effects increase the probability of observing corner solutions. This effect is significant for expenditure on both firewood and fuel oils.

Table 2. Estimation results from 1) an AIDS model with no modifications and 2) an AIDS model adjusted for zero expenditure

	1. No modifications	*2. Adjusted for zero expenditure*	*3. Difference in coefficients*
A. Electricity expenditure			
Constant (α_1)	0.823 ***	0.749 ***	-0.074
Electricity price (γ_{11})	-0.076 ***	-0.052 **	0.025
Fuel oil price (γ_{12})	-0.012 ***	-0.057 ***	-0.045 ***
Firewood price (γ_{13})	0.008 **	-0.004	-0.012 **
Energy budget (β_1)	-0.261 ***	-0.261 ***	0.000
Standard deviation (σ_1)	4.105 ***	4.100 ***	-0.006
B. Fuel oil and paraffin expenditure			
Constant (α_2)	0.069 **	0.586 ***	0.518 ***
Electricity price (γ_{21})	-0.012 *	-0.082 ***	-0.070 ***
Fuel oil price (γ_{22})	0.016 ***	-0.035 ***	-0.050 ***
Firewood price (γ_{23})	-0.003 ***	-0.009 ***	-0.006 **
Energy budget (β_2)	-0.016 ***	-0.106 ***	-0.090 ***
Standard deviation (σ_2)	1.435 ***	2.990 ***	1.555 ***
Share of zero expenditure (κ_2)		0.280 ***	
Probability of zero expenditures on fuel oil			
Constant (ψ_{20})		0.865 *	
Electricity price (ψ_{21})		-0.011	
Fuel oil price (ψ_{22})		-0.019 ***	
Firewood price (ψ_{23})		0.006 ***	
C. Firewood expenditure			
Constant (α_3)	0.003	0.120 **	0.117 *
Electricity price (γ_{31})	0.012	0.006	-0.005
Fuel oil price (γ_{32})	-0.006 ***	-0.019 ***	-0.013 ***
Firewood price (γ_{33})	0.008 ***	-0.007	-0.015 ***
Energy budget (β_3)	-0.041 ***	-0.075 ***	-0.034 ***
Standard deviation (σ_3)	1.808 ***	2.196 ***	0.388 ***
Share of zero expenditure (κ_3)		0.804 ***	
Probability of zero expenditures on firewood			
Constant (ψ_{30})		0.156	
Electricity price (ψ_{31})		-0.024 ***	
Fuel oil price (ψ_{32})		0.010 ***	
Firewood price (ψ_{33})		0.004 **	

Coefficients marked *, **, or *** are significant at the 10, 5 and 1 per cent level respectively.

The estimation results for the probability of observing corner solutions for fuel oils are somewhat unexpected considering the Kuhn-Tucker condition in equation (2), as we would expect an increase in a cross price and a reduction in the own price to reduce the probability of observing a zero expenditure. One reason for our results may be that households with a central heating system do not have the opportunity to use alternative fuels in the same way as households with e.g. wood stoves. Since the share of central heating systems is highest for households with the opportunity to use fuel oils, this may limit the actual opportunities to use alternative fuels for these households. This leaves only budget effects of a price increase, which lowers the demand for all normal goods and increases the number of zero observations. Another reason why the probability of observing corner solutions for fuel oils decreases with price may be that this model does not contain sufficient heterogeneity to explain what actually happens to fuel oil demand when energy prices change.

5.2 Adjusting for interdependence in household energy expenditure

In the previous section (5.1) we presented the results from an estimation of an AIDS model adjusted for zero expenditure assuming the expenditures on energy goods to be stochastically independent within each household, that is $E\left(\varepsilon_i^h \varepsilon_j^h\right)=0$. In Table 3 we compare these results (part 1 of the table) with an AIDS model adjusted for zero expenditure where the expenditures are assumed to be interdependent, that is $E\left(\varepsilon_i^h \varepsilon_j^h\right)\neq 0$ (see column 2 of Table 3). In the last column, we report the difference between the estimated coefficients in the two models and indicate the level of significance of this difference. Section A of the table gives the results for the electricity expenditure, section B gives the results for the fuel oil and paraffin expenditure and section C gives the results for the firewood expenditure. In section B and C we report the estimated parameters of the expenditure function for households with a positive expenditure and the estimated parameters in the probability of choosing a corner solution.

Assuming the expenditures on energy goods to be stochastically independent within each household does not change any of the coefficients significantly, nor does it change the significance of the coefficients, with one exception. The estimate on γ_{33} becomes significant when assuming interdependence between goods within a household, even if the estimates of the coefficient are equal in the two estimations.

Table 3. Estimation results from 1) an AIDS model adjusted for zero expenditure and 2) an AIDS model adjusted for zero expenditure and interdependence in the expenditures on energy goods

	1. Adjusted for zero expenditure	*2. Adjusted for zero expenditure and interdependence*	*3. Difference of coefficients*
A. Electricity expenditure			
Constant (α_1)	0.749 ***	0.744 ***	-0.005
Electricity price (γ_{11})	-0.052 **	-0.050 **	0.002
Fuel oil price (γ_{12})	-0.057 ***	-0.057 ***	-0.001
Firewood price (γ_{13})	-0.004	-0.005	-0.001
Energy budget (β_1)	-0.261 ***	-0.262 ***	-0.001
Standard deviation (σ_1)	4.100 ***	4.099 ***	-0.001
B. Fuel oil and paraffin expenditure			
Constant (α_2)	0.586 ***	0.591 ***	0.005
Electricity price (γ_{21})	-0.082 ***	-0.081 ***	0.002
Fuel oil price (γ_{22})	-0.035 ***	-0.037 ***	-0.003
Firewood price (γ_{23})	-0.009 ***	-0.010 ***	-0.001
Energy budget (β_2)	-0.106 ***	-0.109 ***	-0.003
Standard deviation (σ_2)	2.990 ***	3.006 ***	0.016
Share of zero expenditure (κ_2)	0.280 ***	0.280 ***	0.000
Probability of corner solution			
Constant (ψ_{20})	0.865 *	0.865 *	0.000
Electricity price (ψ_{21})	-0.011	-0.011	0.000
Fuel oil price (ψ_{22})	-0.019 ***	-0.019 ***	0.000
Firewood price (ψ_{23})	0.006 ***	0.006 ***	0.000
C. Firewood expenditure			
Constant (α_3)	0.120 **	0.131 **	0.011
Electricity price (γ_{31})	0.006	0.004	-0.002
Fuel oil price (γ_{32})	-0.019 ***	-0.018 ***	0.001
Firewood price (γ_{33})	-0.007	-0.007 *	0.000
Energy budget (β_3)	-0.075 ***	-0.077 ***	-0.002
Standard deviation (σ_3)	2.196 ***	2.196 ***	0.000
Share of zero expenditure (κ_3)	0.804 ***	0.804 ***	0.000
Probability of corner solution			
Constant (ψ_{30})	0.156	0.156	0.000
Electricity price (ψ_{31})	-0.024 ***	-0.024 ***	0.000
Fuel oil price (ψ_{32})	0.010 ***	0.010 ***	0.000
Firewood price (ψ_{33})	0.004 **	0.004 **	0.000
D. Correlation coefficients, Likelihood ratio index:			
Electricity and fuel oil (ρ_{12})		-0.00005	
Electricity and firewood (ρ_{13})		0.00858	
Fuel oil and firewood (ρ_{23})		0.16485 ***	

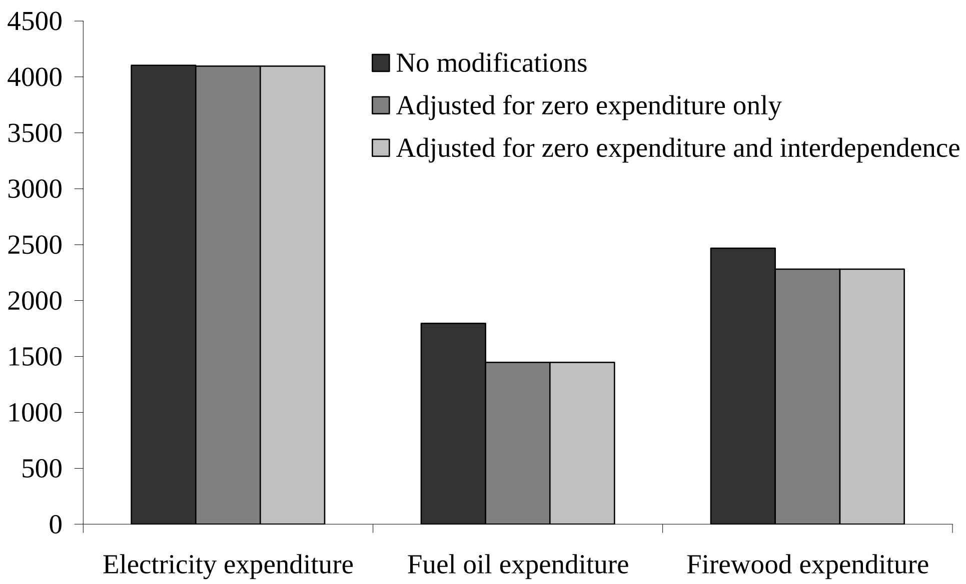
Coefficients marked *, **, or *** are significant at the 10, 5 and 1 per cent level respectively.

The correlation coefficient ρ_{12} is negative, whereas ρ_{13} and ρ_{23} are positive. Only the correlation coefficient between the expenditures on fuel oil and firewood differ significantly from zero. This indicates that the expenditures on fuel oil and firewood are not stochastically independent in this model. Assuming stochastic interdependence within a household does, however, not change any of the estimated coefficients significantly.

5.3 Comparing prediction properties of the models

In order to evaluate different specifications of the model, we compare their ability to describe the data. We focus on two indicators: i) the Mean Squared Error (MSE) and ii) the number of negative predicted budget shares. First, we look at the MSE for different specifications of the model. The MSE is measured as the mean squared difference between predicted expenditure from the estimation and actual expenditure for each household (see also Appendix for more information). The MSE measures how well the estimation fits the data. In Figure 1 we present the MSE for expenditures on electricity, fuel oils and firewood in a model without modifications for zero expenditure, a model adjusted for zero expenditure assuming expenditures to be independent and a model adjusted for zero expenditure assuming interdependence between expenditure on different energy goods.

Figure 1. Mean squared error of expenditures on electricity, fuel oil and firewood in a model without modifications for zero expenditure, a model adjusted for zero expenditure assuming independence and a model adjusted for zero expenditure assuming interdependence



We see from the figure that the MSE of the electricity expenditure is constant across different specifications of the estimation model, whereas the MSEs for fuel oils and firewood expenditures are reduced when using a DH model to adjust the estimations for zero expenditure. This is particularly so for the expenditures on fuel oils, where the share of zero observations in the sample is 84 percent. We also see that assuming stochastic interdependence within a household does not change the fit of the estimated expenditure function as measured by the MSE.

From economic theory, we know that the budget shares should be between zero and one. In our estimations, none of the estimated budget shares exceeded one but some were negative. Thus, we compared the share of negative predictions of budget shares in the three different specifications of the model. In the estimation without any modifications for zero expenditure, 14 percent of the sample had a predicted negative budget share on fuel oils and 3 percent had a negative predicted budget share on firewood. When applying the DH model to adjust for zero expenditure, none of the households were predicted with a negative budget share on any goods. This applies both when assuming stochastic independence and interdependence in the DH model (se also appendix).

6. Concluding remarks

The main aim of this paper has been to estimate household energy consumption, accounting for differences in consumption opportunities and zero expenditure. We apply a version of the DH model to adjust the estimation for zero expenditure either due to limited consumption opportunities or corner solutions. Given the opportunity to consume a good, the choice of corner solution versus a positive expenditure is modeled by a stochastic Kuhn-Tucker optimization problem. The expenditures on energy goods are estimated simultaneously, taking into account that not all households have the opportunity to consume all energy goods. Furthermore, we assume that the stochastic components in the model may be interdependent between energy goods within each household. We compare the results from estimating three alternative specifications; a) an AIDS model without modifications for zero expenditure or stochastic interdependence, b) a multivariate DH model where expenditures are assumed to be independent and c) a multivariate DH model where expenditures are assumed to be interdependent. This comparison is done to test the effects on the estimated coefficients and prediction properties of the alternative specifications of the model.

Comparing the results, we find that adjusting the estimations for zero expenditure affects several coefficients significantly compared to the model with no modifications. This is in particular so in the expenditure functions for fuel oils and firewood, since zero expenditure is common for these energy goods. All households in this sample have a positive expenditure on electricity and thus there are no effects of the modifications on the estimated demand function for electricity. Assuming stochastic interdependence in addition to adjusting for zero expenditure gives approximately the same estimated coefficients as in the model adjusted for zero expenditure assuming stochastic independence, as none of the estimated coefficients change significantly. We find that one of the three correlation coefficients differs significantly from zero. Furthermore, we find that adjusting the estimation for zero expenditure improves the fit of the estimation compared to a model with no modifications, both with respect to the MSE and the number of negative predicted budget shares. Assuming stochastic interdependence, in addition to adjusting for zero expenditure, does not improve the fit.

In conclusion, it seems that adjusting the estimations for zero expenditure has a major impact on the results when the data contains a considerable number of zero observations. Furthermore, we find that assuming stochastic interdependence in addition to adjusting for zero expenditure does not give different results from the model only adjusted for zero expenditure, neither with respect to the estimated coefficients nor the fit. Assuming stochastic interdependence complicates the analysis considerable, since we no longer may use a single equation double hurdle model on each expenditure function to adjust for zero expenditure. Our results imply that this assumption may be unnecessary.

In our model, the only heterogeneity across households included in the expenditure functions is the opportunity to consume various energy good. In reality, household expenditure on energy goods is heterogeneous for several reasons, e.g. due to differences in the number and age of household members, house type, etc. across households. The aim of this analysis has been to find a way of modifying the estimation of household energy expenditures for zero observations. In order to describe the heterogeneity of Norwegian residential energy consumption, we need to include more characteristics of the household in the estimations to describe the heterogeneity across households. This will be the focus in our future research on this topic.

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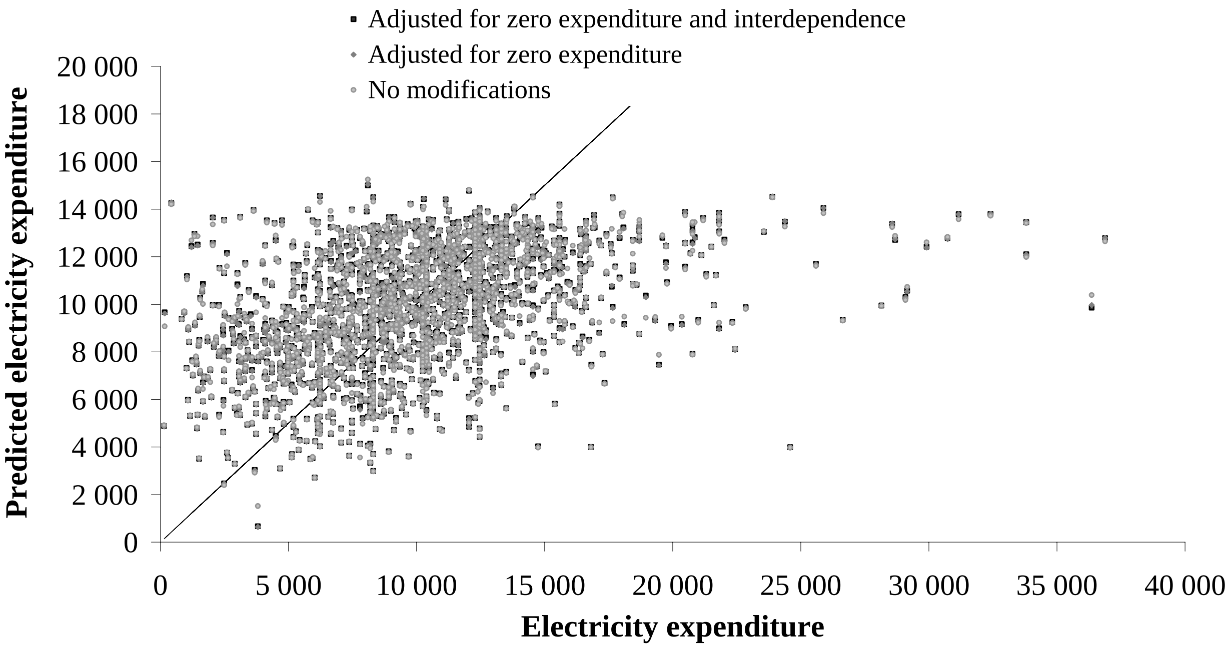
In Figures A1 - A3 we plot predicted expenditures for electricity, firewood and fuel oils for all households in the sample and the line representing equality between predicted expenditure and actual expenditure. The predicted expenditures are calculated using the results from an estimation without any modifications, an estimation adjusted for zero expenditure and an estimation adjusted for zero expenditure and interdependence between goods within each household.

Figure A1. Predicted electricity expenditures versus actual electricity expenditure. NOK



Comparing the fit of different estimation models for the electricity expenditure, it is difficult to see if any model fit the data better. For the expenditures on firewood and fuel oils, it seems that the models adjusted for zero expenditure perform better than the estimation with no adjustments. It is, however, impossible to separate the predictions of the model adjusting for both zero expenditure and interdependence from the predictions of the model adjusting for zero expenditure only.

We see from all the figures that prices and total expenditures do not explain much variation in household energy consumption. This is particularly so for households with high or low expenditures. The predictions fit the data best for the electricity consumption. In order to explain more of the variation in Norwegian residential energy consumption, we need to include more heterogeneity in characteristics of the household and residence in the model.

Figure A2. Predicted firewood expenditures versus actual firewood expenditure. NOK

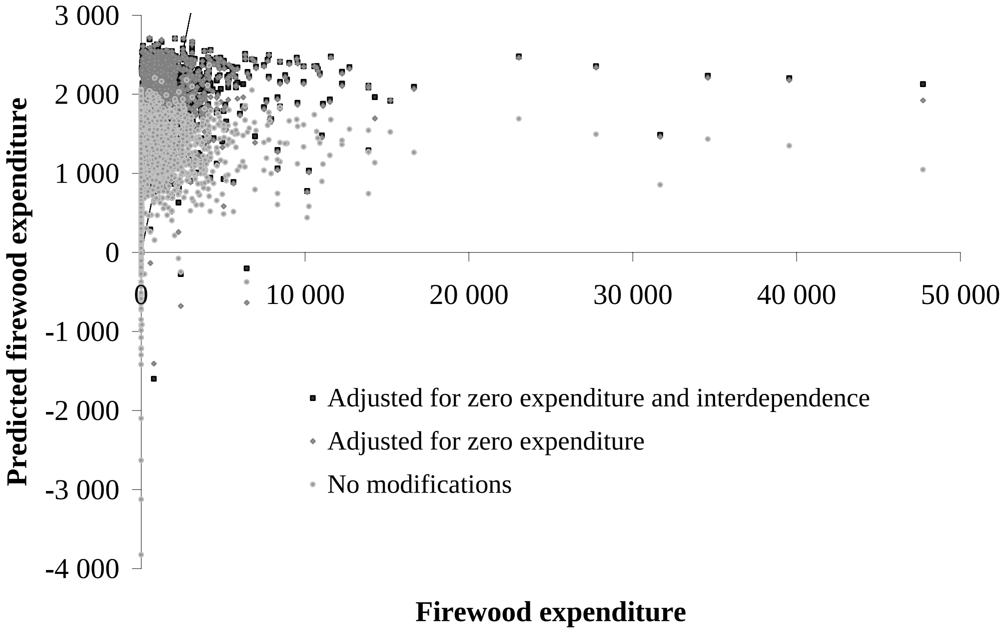
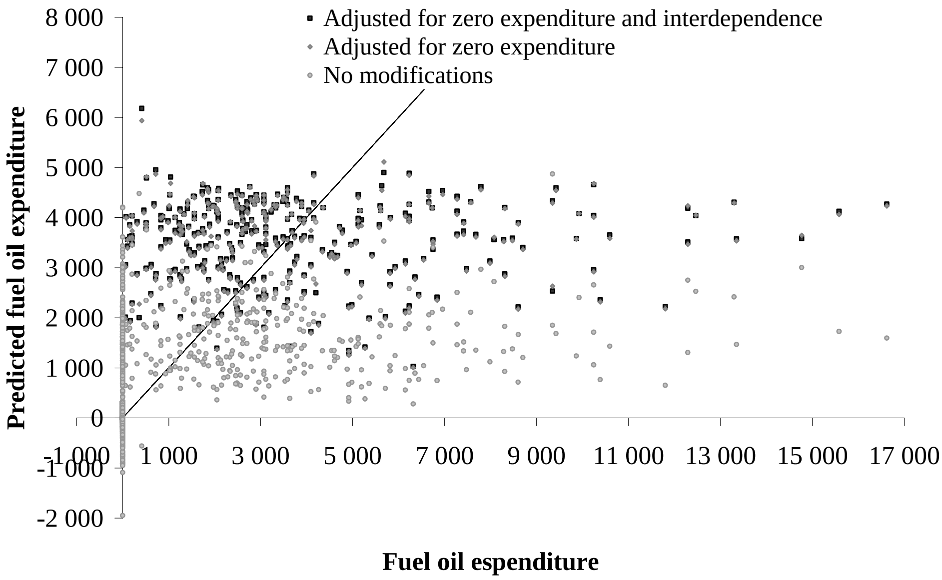


Figure A3. Predicted expenditure on fuel oils versus actual expenditure on fuel oils. NOK



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