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## **Aggregate Industry Behaviour in a Monopolistic Competition Model with Heterogeneous Firms**

**Abstract:**

The paper analyses how a tractable representation of productivity heterogeneity among firms modifies the standard Dixit-Stiglitz (DS) model of monopolistic competition. The properties of the asymmetric model are explored by comparative statics analysis. The equilibrium adjustments of industry aggregates, such as the number of firms, output and the productivity of variable inputs are compared with the corresponding adjustments in the symmetric model. The analysis thereby clarifies when and why the standard DS model may provide a misleading picture of the aggregate industry behaviour. A by-product of the paper is to demonstrate that a more realistic description of a basic aspect of technology heterogeneity can be taken into account at a relatively low cost in terms of reduced tractability.

**Keywords:** Monopolistic competition, productivity heterogeneity, aggregation

**JEL classification:** L11, L13, L16

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# 1. Introduction

*"A universal adoption of the assumption of monopoly must have very destructive consequences for economic theory."* (John Hicks (1939, p. 83)).

*"...the theory of monopolistic competition has had virtually no impact on the theory of international trade."* (Harry G. Johnson (1967, p. 203)).

The judgements and predictions expressed in these quotations could hardly have been more wrong. The formal model of Chamberlinian monopolistic competition<sup>1</sup> introduced in Dixit and Stiglitz (1977), has become a workhorse in several branches in economic theory, including international trade, economic growth and macroeconomics. The Dixit-Stiglitz (DS) model has proved useful to a degree that has justified the term "The monopolistic competition revolution".<sup>2</sup> A survey of past and new contributions to this revolution is given in Brakman and Heijdra (2003). The following quotation from Neary (2000, p.2) is probably representative for the opinion of most economists of today: *"The widespread adoption of the Dixit-Stiglitz approach to monopolistic competition has had hugely positive consequences for many branches of economic theory and especially for international trade theory."*

Tractability is a major reason for the popularity of the DS model. Tractability is a result of simplifying assumptions concerning functional forms, free entry, no perceived interdependence between firms, and that firms are identical. The price of simplifying assumptions is that the clear results may be misleading except for a narrow range of questions. Montagna (1995) goes a step further when she considers the assumption of identical production functions in this model to be inconsistent with the interpretation of the Chamberlin's model. Also Stigler (1949) argued that product differentiation is unlikely to exist without non-uniform costs. However, all models are "wrong". The good model maximises relevant insight to specific problems subject to several constraints, including the trade-off between tractability and realism.

This paper moves along the trade-off between realism and tractability with respect to the technology assumptions made in the DS model. Rather than assuming that all firms in the industry, both the active

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<sup>1</sup> See Chamberlin (1933).

<sup>2</sup> Dixit (2000) reports the number of appearances of the phrase "monopolistic competition" in the title and abstracts of articles in the database Econ-lit. Appearances in titles increased from less than 20 in the period 1969-1979 to 100 in 1990-1994. Whereas the number of appearances in titles fell to 69 in 1995-1999, the total number of appearances in both titles and abstracts increased from 176 in 1990-1994 to 211 in 1995-1999.

ones and the potential entrants, have identical technologies, the DS model is extended to incorporate firm heterogeneity with respect to the productivity of variable inputs. The resulting asymmetric model is used to analyse the equilibrium adjustments of aggregate industry variables to changes in exogenous variables. Moreover, these results are systematically compared with the corresponding ones obtained in the special case of identical firm and symmetric equilibrium. Thus, the robustness of the properties of the symmetric DS model is examined with respect to an extension, which clearly improves the degree of realism of the model. Empirical studies provide massive evidence of large and persistent productivity differentials between firms even within narrowly defined industries, see e.g. Baily, Hulten and Campbell (1992), Sutton (1996), Klette (1999) and Klette and Mathiassen (1995, 1996). One of the purposes of the paper is to demonstrate that the cost of taking into account some basic characteristics of productivity heterogeneity can be relatively low.

A higher degree of realism in a model does not necessarily provide increased insight into interesting problems. Is a more realistic description of productivity heterogeneity likely to generate important new insights? There are some reasons to think so, even *ex ante* a formal analysis. The DS model has proved particularly convenient in analyses of aggregate industry responses, e.g. in the context of international trade. The relationship between aggregate industry behaviour and the behaviour of the individual firms was at the heart of the seminal works of e.g. Houthakker (1955-56), Salter (1960), Johansen (1972) and Sato (1975)<sup>3</sup>. These works clarified the distinction and correspondence between micro and macro production functions, in particular why and how productivity heterogeneity was crucial for the scale- and substitution properties of the aggregate production function. For example, both Johansen (1972) and Houthakker (1955-56) formalised the old insight that the combination of productivity heterogeneity and free entry gives rise to decreasing returns to scale of the industry production function. In studies of industry or trade policies, one would intuitively expect that reduced assistance to firms, would imply a smaller reduction of industry output when the contraction takes place through exit of the least efficient firms, compared to a situation where all firms are assumed to be identical.

In the models used by Johansen and Houthakker, respectively, as well as in the symmetric DS model, the output of the individual firms is fixed, i.e. independent of demand side variables. In Johansen (1972), Houthakker (1955-56) the output of each active firm is given by an exogenous capacity constraint. In the symmetric DS model firm size is fixed by tastes and technology, so that adjustments of industry size to changes in demand come about through changes in the number of firms. As will be demonstrated in the subsequent analysis, the asymmetric DS model provides a richer story of how the

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<sup>3</sup> Also in Marshall's theory of value, as laid out in Frisch (1950), the argument for assuming decreasing returns to scale at the aggregate industry level relies on the existence of productivity differentials between firms combined with a selection mechanism, which ensures that only profitable firms are active.

size of the firm and the industry as a whole is determined, involving mechanisms on both the demand and supply side.

Montagna (1995), Leahy and Montagna (1997, 1998), and Molana and Montagna (2000) stand out as exceptions from the tradition of assuming identical production functions of the individual firms in the DS model. The model framework used in the present paper differs from the one used in these papers by imposing another structure on the inter-firm productivity differentials, yielding a return in terms of analytical tractability. Another difference lies in the purpose of the papers; the present one performs a systematic comparison between the symmetric DS model and the asymmetric DS model resulting from a particular structure of productivity heterogeneity, thereby examining how robust the properties of the DS-model is with respect to productivity heterogeneity. The paper is related to Holmøy and Hægeland (1997, 2000) who explored the determinants of aggregate industry productivity within an asymmetric DS model.<sup>4</sup> The model used in the present paper is less general than the one used in Holmøy and Hægeland (1997, 2000) by sticking to the restrictive assumptions in the standard DS-model, i.e. i) constant returns to scale in variable inputs, and ii) Cobb-Douglas preferences over the composite industry good and the aggregate of all other goods. The return is increased transparency of the most important mechanisms, and the asymmetric DS model becomes comparable with its symmetric counterpart.

The rest of the paper is organised as follows: Section 2 presents the asymmetric DS model and derives closed form solutions for aggregate industry variables including output, costs, the price index of the differentiated industry product, and the number of active firms. These solutions are compared to the corresponding solutions obtained in the standard symmetric version of the model. Section 3 presents and interprets aggregate industry responses to changes in exogenous variables including shifts in both technology parameters and policy measures. The model makes it possible to compare uniform cost shifts in all firms with non-uniform shifts, which may reduce the productivity gaps between firms. Where meaningful, the comparative statics results are compared with the analogous results in the symmetric version of the model. Section 4 concludes.

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<sup>4</sup> A more general version of the model analysed in this paper has been calibrated to data for several Norwegian manufacturing and service industries and implemented in a large CGE-model of the Norwegian economy. See e.g. Fæhn and Holmøy (2000) for an example of an applied trade policy analysis based on this model.

## 2. The model

The point of departure is the standard DS-model of monopolistic competition between a large group of firms. The “differentiated products” industry consists of  $n$  firms, each one producing a product being a close but imperfect substitute for the products supplied by the other firms in the industry. A representative consumer represents the demand side. His preference structure is separable: The  $n$  different goods enter symmetrically into a linearly homogenous Spence-Dixit-Stiglitz (SDS) utility function. The corresponding sub-utility level is a measure of the real consumption of the differentiated product, which enters a top level Cobb-Douglas utility function together with a quantity index of all other goods. The budget share of the differentiated product will then be constant, and the direct price elasticity of the differentiated industry product equals -1. Since the total expenditure is exogenous, expenditure spent on the differentiated good will be exogenous in this partial model. This assumption is made in order to focus on the changes *within* the industry, whereas the total industry output will be exogenous when measured in units of the other good. Holmøy and Hægeland (1997, 2000) consider a more general preference structure.

Equilibrium in each variety market requires

$$X_i = \left( \frac{P_i}{P} \right)^{-\sigma} \frac{Y}{P}, \quad (1)$$

where  $X_i$  is the output of firm  $i \in [0, n]$ ,  $P_i$  is the price of the variety produced by the  $i$ 'th firm,  $\sigma > 1$  is the elasticity of substitution between the varieties constituting the differentiated product,  $Y$  is the exogenous expenses allocated to the differentiated industry product, and  $P$  is the ideal price index for this product. For analytical convenience the index  $i$ , indicating firms and varieties, is treated as a continuous variable. The form of the price index  $P$  follows from the SDS-preferences:

$$P = \left( \int_0^n P_i^{1-\sigma} di \right)^{1/(1-\sigma)}. \quad (2)$$

The cost function of firm  $i$  is

$$C(X_i) = c_i X_i + F,$$

where  $F$  is a uniform fixed cost and  $c_i$  is the constant firm specific marginal cost.

Firms are ranked according to productivity of the variable input so that firm  $\theta$  is the most efficient one. We assume a constant relative productivity differential between any two adjacent firms, i.e.

$$\dot{c}_i = t c_i \Leftrightarrow c_i = c e^{t i}, \quad t > 0, \quad (3)$$

where  $-t$  is proportional to the relative productivity differential between adjacent firms.  $c_0 = c$  is exogenous.<sup>5</sup> In the special case of homogeneous firms,  $c_i = c$  as in the standard DS-model.<sup>6</sup> As in the DS-model,  $n$  is assumed to be large enough to make the market share of even the largest firm insignificant. Thus, each firm is a price index taker in the sense that it neglects the impact of its own price setting on the price index  $P$ . The perceived demand elasticity is equal to  $-\sigma$ , and maximisation of (operating) profits then implies the familiar mark-up price setting rule

$$P_i = m c_i, \quad (4)$$

where  $m = \sigma/(\sigma - 1)$  is the mark up factor. Maximised operating profits in firm  $i$  becomes

$$\pi_i = (m - 1) c_i X_i. \quad (5)$$

In the equilibrium where incentives for firms to enter or exit the industry are eliminated, the following condition must hold for the least productive active firm:

$$\pi_n = F. \quad (6)$$

Subsequently, firm  $n$  will be referred to as the marginal firm. The model (1) - (6) determines the endogenous variables  $X_i$ ,  $c_i$ ,  $P_i$ ,  $P$ ,  $\pi_i$ , and  $n$ .

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<sup>5</sup> As long as the heterogeneity applies to private cost, the cost differentials may also be interpreted as a result of a particular uneven distribution of industry aid from the government. Often uneven subsidisation of firms is used to modify the productivity differentials, i.e. to make inefficient firms more profitable. A special case occurs when the subsidisation exactly neutralises the productivity differentials, so that the private marginal costs to become equal in all  $n$  firms. The equilibrium will be symmetric in the sense that all  $n$  firms will produce identical quantities and charge the same price. In the case where uneven subsidisation overcompensates the productivity differentials, the private variable cost will be a decreasing function of the number of the firm, which is based on productivity ranking. Ordering based on ascending private costs will be opposite to the ordering based on ascending social costs. In the following this possibility will be disregarded.

<sup>6</sup> The level of  $c$  in the symmetric case could differ from  $c_\theta$ . For example it could be equal to the average variable unit cost for the industry. This would of course affect the solution of the symmetric model. However, as long as  $c$  is exogenous, the elasticities with respect to exogenous changes would be unaffected. The assumption  $c = c_\theta$  in the symmetric case should therefore be regarded as a convenient normalisation.

Aggregate output and variable costs are by definition respectively  $X = \int_0^n X_i di$  and

$C = \int_0^n C_i di = \int_0^n c_i X_i di$ . The following steps derives closed form solutions for the aggregates  $P$ ,  $X$  and  $C$ . By inserting (3) and (4) into (2), the price index can be expressed as

$$P = mc \left[ \frac{1 - e^{(1-\sigma)t}}{t(\sigma-1)} \right]^{1/(1-\sigma)}. \quad (7a)$$

In the *symmetric* model of homogeneous firms  $t = 0$ , and the price index takes the well known form

$$P = mc n^{1/(1-\sigma)}. \quad (7b)$$

Inserting (7a) into (1) output from firm  $i$  can be written

$$X_i = \left[ \frac{Yt(\sigma-1)}{mc(1 - e^{(1-\sigma)t})} \right] e^{-\sigma i} \equiv X_0 e^{-\sigma i}, \quad (8a)$$

since the term in the square bracket equals output from the most efficient firm. In the symmetric case the corresponding solution takes the form

$$X_i = \bar{X} = \frac{Y}{mc n}. \quad (8b)$$

The equilibrium solution for  $n$  is found by inserting (2), (5), (7a) and (8a) into (6). One then obtains the closed form solution for  $n$ :

$$0 < e^{(1-\sigma)t} \equiv f(n) = \left( 1 + \frac{tY}{mF} \right)^{-1} \equiv A < 1 \Leftrightarrow n = \ln A / (1-\sigma)t. \quad (9a)$$

In the subsequent derivations of the closed form solutions for the industry aggregates, it is more convenient to work with the transform  $f(n)$  than with the solution for  $n$ . In the symmetric case the number of active firms becomes

$$\bar{\pi} = F \Leftrightarrow (m-1)c\bar{X} = F \Leftrightarrow n = \frac{Y}{F\sigma}, \quad (9b)$$

where the expression for the optimal mark-up factor has been utilised. By using (9a) to eliminate  $n$  in (7a), the solution for  $P$  becomes

$$P = mc \left[ \frac{(\sigma - 1)t}{1 - A} \right]^{1/(\sigma - 1)} = mc \left[ \frac{(\sigma - 1)(mF + tY)}{Y} \right]^{1/(\sigma - 1)}. \quad (10a)$$

The corresponding solution in the symmetric reference case becomes

$$P = mc \left( \frac{\sigma F}{Y} \right)^{1/(\sigma - 1)}. \quad (10b)$$

Integrating output over the equilibrium number of firms yields the aggregate industry production in the asymmetric case

$$X = \int_0^n X_i di = \left[ \frac{Yt(\sigma - 1)}{mc(1 - e^{-(1-\sigma)tn})} \right] \int_0^n e^{-\sigma i} di = \frac{Y(1 - e^{-\sigma n})}{m^2 c(1 - e^{-(\sigma - 1)tn})}.$$

By using (9a), the solution for  $X$  can be written more compactly as

$$X = \frac{Y}{m^2 c} \left( \frac{1 - A^m}{1 - A} \right). \quad (11a)$$

The corresponding solution in the symmetric reference case is

$$X = n\bar{X} = \frac{Y}{mc}. \quad (11b)$$

Independent of the degree of asymmetry, the aggregate variable costs will always equal

$$C = \int_0^n c_i X_i di = \frac{Y}{m}, \quad (12)$$

which equals the total consumer expenditure on the composite industry good evaluated at marginal producer costs.

### 3. Comparative statics

In order to analyse the importance of extending the DS-model by allowing firm heterogeneity with respect to the productivity of variable inputs, this section compares how the industry aggregates output, input, the price index for the composite industry product and the number of active firms, respond in the symmetric and the asymmetric model, respectively, to changes in the exogenous technology and demand parameters specified in the model set up in Section 2, i.e.  $c$ ,  $t$ ,  $F$ ,  $Y$  and  $m$  (or  $\sigma$ ). The results of the comparative statics analysis are summarised in Table 1, 2 and 3 in terms of equilibrium elasticities of the number of active firms, aggregate output and output from the most efficient firm,  $X_0$ .

#### *Effects on the number of active firms*

In the asymmetric model, the most complex changes in industry aggregates come through changes in  $n$ . The partial elasticities of  $n$  with respect to the relevant exogenous variables, and their order of magnitude, are summarised in Table 1 below. It has been utilised that  $-1 < (1-A)/\ln(A) < 0$ , which is shown in Appendix 1.

**Table 1. Partial equilibrium elasticities of the number of active firms. AS = Asymmetric model, S = Symmetric model**

<i>Model</i>	$c$	$m$	$Y/F$	$t$
AS	0	$\frac{1-A}{\ln(A)} + \sigma$	$\frac{A-1}{\ln(A)}$	$-1 < \frac{A-1}{\ln(A)} - 1 < 0$
AS		$\sigma - 1 < \text{El}_m n < \sigma$	$0 < \text{El}_{Y/F} n < 1$	$-1 < \text{El}_t n < 0$
S	0	$\sigma$	1	-

#### *A proportional productivity shift in all firms*

$c$  is irrelevant for the equilibrium number of firms, irrespective of the degree of heterogeneity. In both model the following logic applies: Due to mark-up pricing, an increase in  $c$  implies higher operating profits for a given output level. Since all prices of the *ex ante* available products rise by the same proportion,  $P$  will increase by the same proportion. The composition of the composite industry good will not change. As a consequence of the assumption of Cobb-Douglas preferences at the upper decision level, the direct price elasticity of the composite industry good is equal to -1. Thus, for all firms the profit effect of reduced output exactly outweighs the increase in the profit per unit of output, and there is no incentive to enter or exit. If the demand for this composite were more elastic, the operating profits for the marginal firm would be decreasing in  $c$ . Hence, the equilibrium number of firms would also be a decreasing function of the product of these variables, see Holmøy and Hægeland (1997) for a further discussion.

### *Shift in demand and fixed costs*

It follows directly from (9a) and (9b) that an increase in  $Y$  has the same relative effect on  $n$  as a proportional reduction in  $F$ . A partial increase of  $Y$  by 1 percent raise the demand function facing all firms by 1 percent. An increase in output by 1 percent from each *ex ante* existing firm would balance the product markets, but the profits earned by the marginal firms would exceed the fixed cost. Entry occurs until the *new* marginal firm earns profit equal to the constant fixed cost. (9b) shows that the entry/exit condition in the *symmetric model* determines the uniform output level in all firms independent of demand parameters. Thus, increased demand is not met by more output in the existing firms, only by output from the new identical firms. Accordingly,  $n$  is proportional to  $Y/F$  in the symmetric model.

In the *asymmetric model* the expansion of  $n$  is under proportionate and equal to  $0 < (A-1)/\ln A < 1$  in relative terms. The reason why the expansion is smaller than in the symmetric case is that the new firms successively will set the price at a higher level than the pre-existing firms, because they become increasingly inefficient. Entry will therefore induce some substitution in favour of the existing products. Thus, contrary to the symmetric model, the increase in demand is met not only by output from new entrants, but also by higher output in the intra-marginal firms. The elasticity of the output from the most efficient firm with respect to  $Y$  can be shown to equal  $0 < 1 - A < 1$ , see Table 3.

A second effect is caused by the additional utility experienced from an expansion of the product spectre. In the model this effect is accounted for by a reduction in the price index,  $P$ , which, *cet. par.*, raises the demand facing the industry as a whole. Qualitatively, this demand effect works in the same way in this asymmetric model as in the symmetric one, but the quantitative impact is smaller in the asymmetric case. *Cet.par.*, deviation from a uniform distribution of variety consumption has a negative impact on the utility, i.e. a positive impact on  $P$ . The increase in  $Y$ , combined with entry, reinforces the distribution of the industry output in favour of the most efficient firms. The resulting positive effect on  $P$  modifies the expansion of firms compared to the symmetric model.

### *Changes in heterogeneity*

Changes in heterogeneity are captured by changes in  $t$ . An increase in  $t$  raises unit costs in all but the most efficient firm. Due to mark-up pricing, substitution in demand will increase the output from the most efficient firm, whereas output from the other *ex ante* existing firms goes down. For a given number of firms,  $X_0$  increases proportionally to the increase in  $t$ , cf. (8a). The *ex ante* marginal firms become less profitable and exit. However, the analytical results reported in Table 1 show that the reduction in  $n$  is not large enough to prevent a reduction in the product  $nt$ , which shows up in the exponent in  $f(n)$  defined in (9a). If, hypothetically, the equilibrium value of  $nt$  were invariant to

exogenous changes in the model, the profit condition (6) would imply that also the output of the marginal firm would be invariant to changes in  $t$ . This is not the case in the asymmetric model. Instead, output is higher in all *ex post* profitable firms. Since profit increases in output, the equilibrium elasticity of  $n$  with respect to  $t$  is greater than -1.

### *Preferences for variety*

A preference for variety is captured by the elasticity of substitution  $\sigma$ . The lower is  $\sigma$  the less perfect can products from different firms replace each other. Thus, the utility obtained from a given degree of variety, i.e. by a given number of firms and products in this model, is a negative function of  $\sigma$ . By duality, this is captured by  $P$  being an increasing function of  $\sigma$ , for given prices of the individual varieties. As pointed out above, this model implies a direct link between the optimal mark-up  $m$  and the preference parameter:  $m = \sigma/(\sigma-1)$ . The elasticity of  $m$  with respect to  $\sigma$  is  $1-m < 0$ . An increase in  $\sigma$  implies a higher perceived price elasticity facing each monopolistic producer. Each firm will therefore reduce its price by setting a lower mark-up. In the symmetric model the latter effect is stronger than the love-of-variety effect, so there is a negative relationship between  $P$  and  $\sigma$ .

A shift in preferences by which variety is more appreciated is captured by a lower  $\sigma$ . In the tables the reduction in  $\sigma$  has been dimensioned to  $1/(1-m)$  percent, so that the increase in  $m$  equals 1 percent. A higher mark-up leads to entry because profit increases for *ex ante* profitable firms. In the *symmetric model* the elasticity of  $n$  with respect to  $m$  becomes  $m/(m-1) = \sigma$ , which may be large in the LGMC case. The number of entrants is smaller in the *asymmetric model* than in the symmetric one; the elasticity of  $n$  with respect to  $m$  lies in the interval  $(\sigma-1, \sigma)$ . The modification of this elasticity, caused by heterogeneity, is therefore small when  $\sigma$  is large. The main reason for this modification is the same as provided in the case of an increase in  $Y$ : New entrants charge a higher price than do the existing firms. The resulting substitution in favour of existing products implies a negative drag on their profitability.

### *Aggregate industry output*

Logarithmic differentiation of the closed form expression for aggregate industry output yields the equilibrium elasticities reported in Table 2. In the asymmetric model, some of the elasticities depend on the variable  $\Delta \equiv A/(1-A) - mA^m/(1-A^m)$ , which occurs after logarithmic differentiation of the function  $(1-A^m)/(1-A)$  entering (11a). In Appendix 2 it is shown that  $\Delta > 0$ , and how the properties of this function in  $A$  and  $m$  can be utilised in order to constrain the orders of magnitude of the equilibrium elasticities of  $X$ .

**Table 2. Partial equilibrium elasticities of aggregate industry output. AS = Asymmetric model, S = Symmetric model**

<i>Model</i>	<i>m</i>	<i>c</i>	<i>F</i>	<i>Y</i>	<i>t</i>
AS	$-2+\Delta(1-A)-A^m \ln(A^m)/(1-A^m)$	-1	$\Delta(1-A)$	$1+\Delta(A-1)$	$\Delta(A-1)$
AS	$-2 < -2+A < \text{El}_m X < -1$		$0 < \text{El}_F X < A < 1$	$0 < 1-A < \text{El}_Y X < 1$	$-1 < -A < \text{El}_t < 0$
S	-1	-1	0	1	-

*A proportional productivity shift in all firms*

Mark-up pricing implies that an increase in  $c$  by 1 percent is shifted forward to a 1 percent increase in the prices of all existing varieties and  $P$ . Thus, there is no substitution within the industry. As noted above, the assumption of Cobb-Douglas preferences implies no entry or exit, and the demand for the composite industry good, as well as each variety, falls by 1 percent. These mechanisms apply to the symmetric as well as to the asymmetric model. Measured in relative terms, the impact of  $c$  on  $X$  is independent of the degree of heterogeneity.

*Shift in demand*

In the *symmetric model*  $X$  is proportional to  $Y$ , cf. (11b). Given the assumptions above, the expansion of the industry output caused by an increase in  $Y$  is brought about by entry, whereas the size of the representative firm is constant.

In the *asymmetric model*, however, each firm producing *ex ante* will increase its output. From the derivations in appendix 2 it follows that the equilibrium elasticity of  $X$  with respect to  $Y$ ,  $\text{El}_Y X$ , becomes smaller when firms become different with respect to productivity, cf. Table 2. There are three reasons why  $\text{El}_Y X$  is smaller in the asymmetric than in the symmetric model: First, the number of entrants is smaller since their productivity is successively decreasing. Second, the output from the new entrants is successively decreasing from the level produced by the *ex ante* marginal firm. Third, the spread of the budget shares increases as successively less efficient and smaller firms enter the industry. Since preferences are symmetric, this implies a negative modification of the increase in  $P$  caused by an extended product spectre. Consequently, the decrease in  $P$  is smaller than in the symmetric case, which implies a negative modification of aggregate demand.

*Shift in fixed costs*

An increase in the fixed cost affects  $X$  only through a reduction in the number of firms. Irrespective of the degree of symmetry  $F$  affects  $X$  through the term  $Y/F$ . Thus, in relative terms the change in  $X$  from an increase in  $F$  is symmetric to the effects *working through entry*, when  $Y$  increases. The logic in the

preceding paragraph therefore applies when the equilibrium elasticity  $El_{FX}$  is to be explained. In the *symmetric model* this elasticity equals -1.

The absolute value of  $El_{FX}$  is smaller in the *asymmetric model* because the number of exiting firms is smaller, because the exiting firms represent relatively small output shares, and because exit reduces the spread in the budget shares. The latter effect implies that the increase in  $P$ , due to a more limited product spectre, is modified downwards compared to the symmetric model. Consequently, the decline in the degree of heterogeneity contributes to raise aggregate demand ( $Y/P$ ).

### *Changes in heterogeneity*

In relative terms  $t$  and  $F$  are shown to have a symmetric effect on  $X$ . This result is perhaps not intuitively obvious since increases in  $t$  and  $F$  are associated with costs and entry barriers. Let us decompose the impact of  $t$  on  $X$  into effects on output from the most efficient firm,  $X_0$ , the other *ex ante* profitable firms and the output from firms that exit.

A rise in  $t$  raises marginal costs in all but the most efficient firm. Consequently, prices of all but the cheapest variety (number 0) are set higher. The resulting increase in  $P$  implies a reduction of the demand for the differentiated product. In addition, it brings about substitution of demand in favour of the most efficient firm. Since  $\sigma > 1$ , this substitution effect dominates the equilibrium adjustment of  $X_0$ , cf. Table 3. However, the substitution within the industry implies that the ratios  $X_i/X_0$  fall for  $0 < i \leq n$ , which, *cet. par*, contributes to reduce  $X$ . From the derivation of (11a)  $X$  may be written as

$$X = X_0 \int_0^n \frac{X_i}{X_0} di, \text{ where } X_0 = \left[ \frac{Yt(\sigma-1)}{mc(1-A)} \right] \text{ and } \int_0^n \frac{X_i}{X_0} di = \frac{(1-A^m)}{\sigma}. \text{ Neglecting for a moment that } A$$

depends on  $t$  directly and indirectly through  $n$ , it follows that the impacts of  $t$  on  $X_0$  and on  $\int_0^n X_i/X_0 di$  cancel out. Note that when  $tn$  is large (due to small fixed costs and/or a high degree of heterogeneity),  $A$  becomes small as  $\sigma$  is large. Intuitively, this means that the marginal firm has a small share of output,  $X$ , and the budget  $Y$ . More precisely: In the limit the error made by integrating over an infinite number of firms and products rather than an increasing number of firms and products vanishes. This limit corresponds to  $A = 0$ , which would reflect  $F = 0$ , i.e. no entry barrier. In this case neither  $P$  nor  $X$  is affected by a small variation in the number of (infinitesimal) firms.

Thus, marginal changes in  $t$  will not have significant effects on  $X$  if the *ex ante* marginal firm is "small". When this is not the case, the following effects cause a decline in  $X$ : The *ex ante* marginal firm(s) will not be profitable after a rise in  $t$ , because the price sensitivity of the demand for each product prevents the firm from regaining the profits lost by the increase in their variable unit costs.

The exit of firms is necessary until the *ex post* marginal firm earns a profit equal to  $F$ . *Cet. par* exit of course reduces  $X$ . On the other hand, exit implies a less diverse product spectre, which shows up as a negative utility effect through a further increase in  $P$ . Higher  $P$  induces a negative shift in the demand for the composite industry good, and a further substitution in favour of the *ex post* profitable firms, consistent with the exit. The latter effect dominates; logarithmic differentiation of (8a), contingent on  $n$ , verifies that the elasticity of  $X_i$  with respect to  $n$  is negative<sup>7</sup>. On balance, however the direct negative effect on aggregate output from the exit of firms is stronger than the effect of increased output from the *ex post* profitable firms. This explains why  $X$  falls as an equilibrium response to an increase in  $t$ . The fact that the negative elasticity is bounded to be greater than -1 reflects the aforementioned limited impact entry or exit can have on  $X$ .

### *Preferences for variety*

In the symmetric model  $X = Y/mc$ , cf. (11b). From  $m = \sigma/(\sigma - 1)$ , it follows that the elasticity of  $X$  with respect to  $\sigma$  equals  $m - 1 > 0$  in the symmetric case. Thus,  $X$  is a negative function of love of variety. Recall that in Table 2 the negative shift in  $\sigma$  is dimensioned to raise  $m$  by 1 percent. This simple relationship covers the fact that several effects are at work when  $\sigma$  changes, because both the strength of the love of variety and the optimal mark-up are affected by  $\sigma$ . Firstly, a 1 percent reduction of  $\sigma$  implies an increase in the optimal mark-up and a 1 percent increase in  $n$ . Secondly, the output of each firm falls, and the equilibrium effect equals  $-m$  percent. The equilibrium elasticity of  $X$  with respect to  $\sigma$  is therefore  $m - 1$ , which corresponds to the elasticity of  $X$  with respect to  $m$  being  $-1$ .

The equilibrium reduction of  $-m$  percent of the output of each firm can be decomposed into the change in the utility of the composite industry good, given by the indirect utility function  $Y/P$ , and the output of each product per unit of utility. The latter fraction equals  $(mc/P)^{-\sigma} = n^{-m}$  in equilibrium. A fall in  $\sigma$  lowers  $P$  for given values of  $m$  and  $n$ , reflecting the positive utility effect of stronger love of variety. On the other hand, the rise in the optimal  $m$  raises all product prices reducing the utility level, and  $Y/P$  is also reduced as  $n$  shrinks. In relative terms the net effect on  $Y/P$  of a 1 percent fall in  $\sigma$  is an increase equal to  $(m - 1)m \ln n$  percent. The relative change in the equilibrium fraction  $n^{-m}$  is equal to  $-m[(m - 1)\ln n + 1]$  when  $\sigma$  falls by 1 percent. Adding these two contributions gives the  $-m$  percent reduction of the equilibrium output from each firm.<sup>8</sup>

<sup>7</sup> The elasticity of  $X_i$  with respect to  $n$  equals  $(1 - \sigma)mA/(1 - A) < 0$ .

<sup>8</sup> This result can be derived more directly from the non-profit condition, which can be written  $\bar{X} = F/[(m - 1)c]$ . However, this is a reduced form solution for  $\bar{X}$ , which does not shed light on the different mechanisms producing it.

The fact that  $X$  turns out to be independent of  $n$  in the symmetric model reflects that the additional output from entrants is exactly offset by a contraction of output in the *ex ante* profitable firms. This is why  $n$  does not enter (11b). The net effect turns out to be identical to what happens in the much simpler case where  $Y$  is spent on one homogenous good with a price equal to  $mc$ . This property is a consequence of assuming the demand elasticity of the composite industry good to be  $-1$ .

In the *asymmetric* model, things are bit more complex. However, as pointed out above, if the *ex ante* marginal firm is small, a good approximation of the effects on  $X$  can be given by neglecting effects working through entry. Note that for a given number of firms, the marginal firm will be smaller in the asymmetric case than in the symmetric case. It follows from (11a) that approximations to the equilibrium responses in  $X$  can be found from the expression  $X' = Y/m^2c$ , where  $X'$  denotes the approximate solution for  $X$  in the case where the relevant integrals calculated for an infinite number of firms/products. Recall that in the symmetric model  $X = Y/mc$ , so  $X' = X/m$ . Thus, when  $m$  is close to unity, the total industry output will only be slightly smaller in the asymmetric case than in the symmetric one. However, whereas the elasticity of  $X$  with respect to  $m$  is  $-1$  in the symmetric model, it will be close to  $-2$  in the asymmetric model when the marginal firm is of negligible size.

The relative change in  $X$  to a 1 percent fall in  $\sigma$  by 1 percent can be decomposed into the change in output from the most efficient firm,  $X_0$ , and the change in the ratio between the aggregate output from the other firms and  $X_0$ . From (8a)  $X_0 = \left[ \frac{Yt(\sigma - 1)}{mc(1 - e^{(1-\sigma)t})} \right]$  and the latter ratio equals

$X/X_0 = \int_0^1 e^{-\sigma i} di = (1/\sigma)(1 - e^{-\sigma})$ . Neglecting the effects associated with changes in  $e^{-\sigma t}$  and  $e^{(1-\sigma)t}$ , a 1 percent fall in  $\sigma$  raises  $X_0$  by  $2m-1$  percent, whereas  $X/X_0$  decreases by 1 percent. Thus the approximate elasticity of  $X$  with respect to  $\sigma$  equals  $2(m-1)$ , which is equivalent to an elasticity of  $X$  with respect to  $m$  equal to  $-2$ . The ratio  $X/X_0$  corresponds to  $n$  in the symmetric model. Note that  $\sigma$  enters the solution for  $n$  in the symmetric model in exactly the same way as it enters the approximate solution for  $X/X_0$  in the asymmetric model. Comparing  $X_0$  in the asymmetric model with  $\bar{X} = (\sigma - 1)F/c$  in the symmetric model, shows that  $(\sigma-1)$  enters both numerators. However,  $\sigma$  also enters the denominator in  $X_0$  through  $m$ . Thus, a fall in  $\sigma$  has an additional negative impact on  $X_0$  compared to  $\bar{X}$ . The intuitive reason for the difference is that a change in  $\sigma$  has a direct impact on the composition of aggregate output and expenditure when the product prices differ initially; a lower value of  $\sigma$  will cause a redirection away from the most efficient firm.

The modifications brought about by entry are captured by the fraction  $(1 - A^m)/(1 - A)$  in the asymmetric model. It turns out that the increase in  $n$ , together with the fall in  $\sigma$  implies a less negative elasticity of  $X$  with respect to  $m$ . The reason is that the additional output from the entrants is positive. This effect dominates the negative effect caused by the fact that an increase in  $n$  reduces  $P$ , which *cet. par* reduces output from the *ex ante* profitable firms.

**Table 3. Partial equilibrium elasticities of output from the most efficient firm. AS = Asymmetric model, S = Symmetric model**

	$m$	$c$	$F$	$Y$	$t$
AS	$-(1-A+\sigma)$	-1	$A$	$1-A$	$1 - A$
S	$-\sigma$	-1	1	0	-

### *Aggregate productivity of variable inputs*

Since factor prices are exogenous in this model, the productivity of variable inputs may be measured by the unit cost, i.e.  $\bar{C} \equiv \frac{C}{X}$ . In the symmetric model the productivity of variable inputs is by definition equal to  $c$ . When firms differ with respect to productivity, industry expansion through entry of successively less efficient firms implies decreasing returns to scale and a reduced productivity of variable input at the industry level. The following analysis shows, however, that the productivity effect caused by entry/exit is surprisingly limited in the present model. The mechanism producing this result is likely to be quite general as long as a) demand plays a role in the output determination, and b) firms set the product price as a mark-up over marginal costs. Combining (11a) and (12),  $\bar{C}$  can be expressed in terms of exogenous variables by

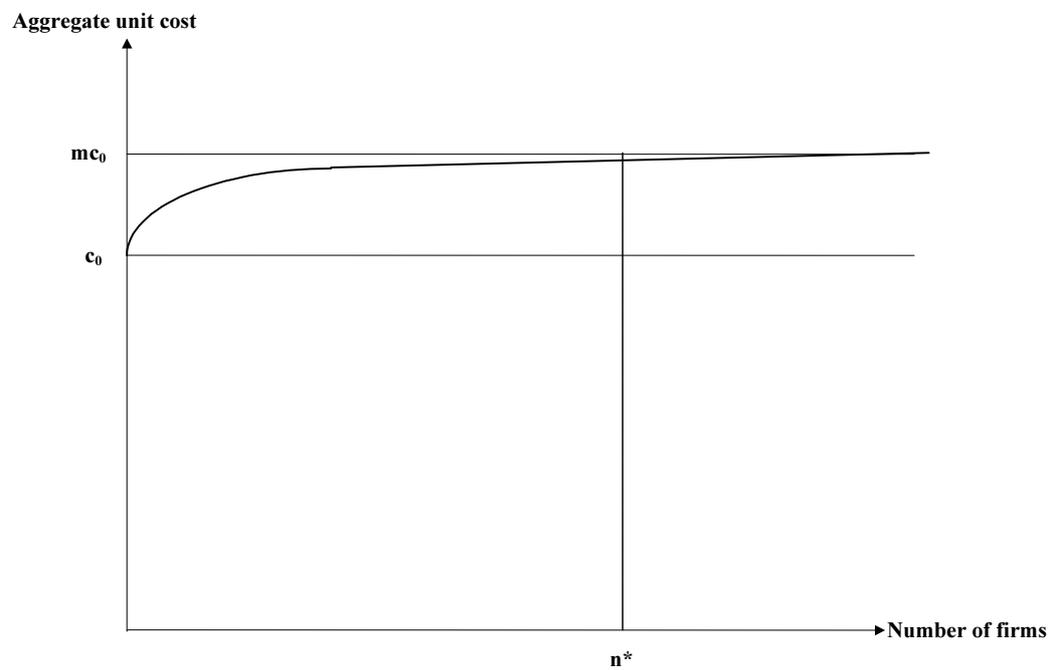
$$\bar{C} = mc \left( \frac{1 - A}{1 - A^m} \right). \quad (13)$$

The impact on  $\bar{C}$  from entry/exit works through  $A$ , where  $0 < A < 1$ . The function  $h(A) = (1 - A)/(1 - A^m)$  is strictly decreasing in  $A$  from its asymptotic value  $h(0) = 1$ , see appendix 2. It converges towards  $1/m$  as  $A$  approaches unity. From (9a) it is clear that the limit  $A = 1$  represents the situation where the fixed cost is infinitely large compared to sales, so no firm can survive.  $A = 0$  corresponds to the opposite limit case where no fixed cost restricts entry and  $n$  is infinitely large. More precisely, entry can only change  $\bar{C}$  within the interval

$$c < \bar{C} < mc. \quad (14)$$

This is illustrated in Figure 1.  $m$  will be relatively close to unity when  $\sigma$  is large, which is typically the case when the assumption of monopolistic competition is appropriate. It follows that all the exogenous variables that affect  $\bar{C}$  through entry, i.e. through changes in  $A$  only, have a limited impact on  $\bar{C}$ . The intuition behind this result is that the weights (output share) of successively less productive entrants in the aggregate industry productivity measure decline at a rate that dominates the rate of increase in variable unit cost. Holmøy and Hægeland (2000) provides a detailed and more general analysis of the determinants of  $\bar{C}$ .

**Figure 1. The relationship between aggregate productivity (unit cost) and the number of firms**



## 4. Concluding remarks

The paper has analysed aggregate industry behaviour within an asymmetric DS-model of monopolistic competition, in which firms differ with respect to the productivity of variable inputs. Although the formal description of productivity heterogeneity is stylised, the model represents a generalisation of the standard symmetric DS-model, which captures important aspects of reality. The paper demonstrates that more realism is gained at a relatively low cost in terms of reduced analytical tractability. The menu of mechanisms that contribute to determine the equilibrium solutions becomes richer, but closed form solutions can still be derived. By comparative statics the properties of the asymmetric DS-model are compared systematically with the corresponding symmetric model. The robustness of the latter is examined with respect to productivity heterogeneity. The partial equilibrium elasticities in the symmetric case become a better approximation to the corresponding elasticities of the asymmetric model, the greater is the number of active firms and the greater is the productivity differentials between firms. On the other hand, the modifications of the marginal properties of the symmetric DS-model become more important the greater is the market share of the marginal firm. Whereas the firm size is invariant to changes in most exogenous variables in the symmetric DS-model, this is not the case in the asymmetric model. Here, expansion of industry output, caused by changes in exogenous variables, will typically take place through both entry and expansion of the *ex ante* existing firms when productivity differ among firms. The endogenous aggregate industry relationship between output and variable input is characterised by decreasing returns to scale. However, the scope for variation in the aggregate industry productivity of variable input is rather limited.

The usefulness of taking firm heterogeneity into account in analyses of questions related to aggregate industry adjustments will be determined through applied and - hopefully - increasingly empirical work. Evidence showing that heterogeneity may play a significant role in the determination of aggregates is rapidly increasing in several fields of economics. The possibilities for providing realistic descriptions of heterogeneity have exploded over the last decade as micro data have become much more easily accessible. Describing and explaining heterogeneity within industries has become a well-established branch in the economics literature. These trends contribute to reduce the cost-benefit ratio of general equilibrium analyses of aggregate industry behaviour, in which representative agents are replaced parametric distributions. The present paper provides one example of how this can be done. Future research will hopefully be able to find parametric distributions that are better compromises between tractability and realism, than both the distribution assumed in this paper and the one assumed in Montagna (1995). Fæhn and Holmøy (2000) applies a large scale CGE-model where the industry behaviour is determined in a sub-model that is somewhat more general than the model analysed above.

A more limited extension of the work presented in this paper would be to introduce heterogeneous assistance to firms as well as productivity heterogeneity. In several cases governments help the least efficient firms through various kinds of industry policies. Such a differential policy is likely to increase the market share of the least productive firms as well as the number of firms. The welfare effects of such policies may be large, and a tractable model of firm heterogeneity is needed to assess them.

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## Appendix 1

In this appendix it is shown that

$$-1 < (1-A)/\ln A < 0. \quad (\text{A.1})$$

Recall from (14) that  $0 < A < 1$ , which implies that  $\ln A < 0$ . Hence the second part of the inequality in (A.1) is trivially fulfilled. In order to show the first inequality,  $\ln A$  is written as the Taylor series:

$$\ln A = y - \frac{y^2}{2} + \frac{y^3}{3} - \dots, \quad (\text{A.2})$$

where  $0 > y = 1 - A > -1$ . It can now be shown that

$$\frac{1-A}{\ln A} + 1 > 0, \quad (\text{A.3})$$

which implies (A.1). Inserting (A.2) into the left hand side of (A.3) yields

$$\begin{aligned} \frac{(1-A)}{\ln A} + 1 &= \frac{y + \ln A}{\ln A} = \frac{-\frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots}{\ln A} \\ &= \frac{-y^2}{\ln A} \left[ \left( \frac{1}{2} - \frac{y}{3} \right) + y^2 \left( \frac{1}{4} - \frac{y}{5} \right) + y^4 \left( \frac{1}{6} - \frac{y}{7} \right) + \dots \right] > 0, \end{aligned} \quad (\text{A.4})$$

because  $\frac{1}{i} - \frac{y}{i+1} > 0$  when  $i$  is a positive integer and  $0 > y > -1$ . This proves (A.1).

## Appendix 2

The closed form solution for aggregate industry output,  $X$ , is proportional to the term

$T \equiv (1 - A^m) / (1 - A)$  where  $0 < A < 1$ . In order to find the partial elasticities of  $X$  with respect to the exogenous variables, one needs the logarithmic differential of  $T$ . A routine calculation yields

$$\hat{T} = \left( \frac{A}{1-A} - \frac{mA^m}{1-A^m} \right) \hat{A} - \frac{mA^m \ln A}{1-A^m} \hat{m} \equiv \Delta(A, m) \hat{A} - g(A, m) \hat{m},$$

where  $\hat{T} = dT/T$  a.s.o. In order to determine the sign and the order of magnitude of the partial elasticities of  $X$ , it is necessary to determine the sign of  $\Delta$  and the characteristics of  $g(A, m)$ .

The sign of  $\Delta$  can be determined by examination of the function

$$f(A, h) = \frac{hA^h}{1-A^h}$$

because  $\Delta$  can then be written

$$\Delta = f(A, 1) - f(A, m).$$

Thus, if  $f$  is a monotonous function of  $h$  in the interval  $[1, m]$ , the sign of  $\Delta$  is determined. The derivative of  $\Delta$  with respect to  $h$  becomes

$$f'_h(A, h) = \frac{a}{(1-a)^2} (1-a + \ln a),$$

where  $0 \leq a \equiv A^h \leq 1$ . Making a Taylor expansion of  $\ln a$  from  $a = 1$ ,  $\ln a$  can be written as

$$\ln a = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \dots,$$

which implies that

$$1 - a + \ln a = -\frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \dots = -\sum_{i=1}^{\infty} (a-1)^{2i} \left( \frac{1}{2i} + \frac{1-a}{2i+1} \right) < 0,$$

since each term in the sum is positive. This proves that  $f'_h(A, h) < 0$  globally, which implies that  $\Delta \geq 0$  and increasing from  $\Delta(A, 1) = 0$ . However,  $\Delta$  converges asymptotically towards the limit  $\Delta(A, \infty) = A/(1 - A)$ . This limit can be found by calculating

$$\lim_{h \rightarrow \infty} f(A, h) = \lim_{h \rightarrow \infty} \frac{h}{A^{-h} - 1} = \frac{\infty}{\infty} = - \lim_{h \rightarrow \infty} \frac{A^h}{\ln A} = 0,$$

where L'Hopitals rule has been employed once. Consequently,  $0 \leq \Delta < A/(1 - A)$ .

Now, turn to the function  $g(A, m)$ . In order to determine which values this function spans, it is convenient to write it as a function of  $a$ :

$$g(A, m) = \frac{a \ln a}{1 - a} = G(a).$$

$G(a)$  is monotonically decreasing since

$$G' = \frac{1 - a + \ln a}{(1 - a)^2} = \frac{f'_h(A, h)}{a} < 0$$

in the interval  $0 \leq a \leq 1$ . To find the interval for the values of  $G$ , first note that employing L'Hopitals rule implies that

$$\lim_{a \rightarrow 0^+} a \ln a = \lim_{a \rightarrow 0^+} \frac{\ln a}{1/a} = \lim_{a \rightarrow 0^+} -\frac{1/a}{1/a^2} = \lim_{a \rightarrow 0^+} -a = 0.$$

This implies that

$$\lim_{a \rightarrow 0^+} \frac{a \ln a}{1 - a} = 0.$$

Moreover, by L'Hopital's rule,

$$\lim_{a \rightarrow 1} \frac{a \ln a}{1 - a} = \lim_{a \rightarrow 1} \frac{1 \cdot \ln a + a(1/a)}{-1} = -1.$$

Hence, the function  $g(A, m)$  varies decreases from 0 to -1 as  $a$  increases from 0 to 1.

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