Discussion Papers No. 342, February 2003
Statistics Norway, Research Department

Erik Biørn, Terje Skjerpen and
Knut R. Wangen

Parametric Aggregation of Random Coefficient Cobb-Douglas Production Functions: Evidence from Manufacturing Industries

Abstract:
A panel data study of parametric aggregation of a production function is presented. A four-factor Cobb-Douglas function with random and jointly normal coefficients and jointly log-normal inputs is used. Since, if the number of micro units is not too small and certain regularity conditions are met, aggregates expressed as arithmetic means can be associated with expectations, we consider conditions ensuring the existence and stability of relationships between expected inputs and expected output and discuss their properties. Existence conditions for and relationships between higher-order moments are considered. An empirical implementation based on panel data for two manufacturing industries gives decomposition and simulation results for expected output and estimates of the aggregate parameters. Illustrations of approximation procedures and aggregation errors are also given.

Keywords: Aggregation. Productivity. Cobb-Douglas. Log-normal distribution. Random coefficients. Panel data.

JEL classification: C23, C43, D21, L11

Acknowledgement: We are grateful to Johan Heldal for helpful and clarifying comments on statistical issues and to Dag Einar Sommervoll for kindly suggesting the proof in Appendix A.

Address: Erik Biørn, University of Oslo, Department of Economics; and Statistics Norway, Research Department. Address: University of Oslo, Department of Economics, P.O. Box 1095 Blindern, 0317 Oslo, Norway. E-mail: erik.biorn@econ.uio.no.

Terje Skjerpen, Statistics Norway, Research Department, P.O. Box 8131 Dep, 0033 Oslo, Norway. E-mail: terje.skjerpen@ssb.no

Knut R. Wangen, Statistics Norway, Research Department, P.O. Box 8131 Dep, 0033 Oslo, Norway. E-mail: knut.reidar.wangen@ssb.no.

1 Introduction

This is an aggregation study of production functions. The production function is usually considered an essentially micro-economic construct, and the existence and stability of a corresponding aggregate function is an issue of considerable interest in macro-economic modelling and research. Jorgenson remarks that "The benefits of an aggregate production model must be weighted against the costs of departures from the highly restrictive assumptions that underly the existence of an aggregate production function" [Jorgenson (1995, p. 76)].[1] Interesting questions from both a theoretical and an empirical point of view are: Which are the most important sources of aggregation bias and instability? Will aggregation by analogy, in which estimated micro parameter values are inserted directly into the macro function, perform satisfactorily?

In this study we use a rather restrictive parametric specification of the 'average' micro technology, based on a four-factor Cobb-Douglas function with random coefficients, *i.e.*, we allow for both a random intercept and random input elasticities. We assume that the random coefficients are jointly normal (Gaussian), and that the inputs follow a multivariate log-normal distribution. The expectation vector and covariance matrix of the random coefficient vector are estimated from unbalanced panel data for two Norwegian manufacturing industries. The validity of log-normality of the inputs is tested and for the most part not rejected. This, in combination with a Cobb-Douglas technology and jointly normal coefficients, allows us to derive interpretable *parametric* expressions for the aggregate production function. Although Cobb-Douglas restricts input substitution rather strongly, and has to some extent been rejected in statistical tests, this property is a distinctive advantage of this functional form against, *e.g.*, Translog or CES.

Properties of relationships aggregated from relationships for micro units depend, in general, on both the functional form(s) in the micro model and properties of the distribution of the micro variables. Customarily, aggregates are expressed as arithmetic means or sums. If the number of micro units is large enough to appeal to a statistical law of large numbers and certain additional statistical regularity conditions are satisfied,

[1] A textbook exposition of theoretical properties of production functions aggregated from neo-classical micro functions is given in Mas-Colell, Whinston and Green (1995, Section 5.E).

we can associate the arithmetic mean with the expectation [cf. Fortin (1991, section 2), Stoker (1993, section 3), Hildenbrand (1998, section 2), and Biørn and Skjerpen (2002, section 2)], which is what we shall do here. However, we will be concerned not only with relationships expressed by means of expectations of the input and output variables of the production function, but also with relationships in *higher-order origo moments*. Thus our paper is in some respects related to Antle (1983), who is concerned with moments of the probability distribution of output.

Under our stochastic assumptions the marginal distribution of output will not be log-normal. We obtain two analytical formulae of the origo moments of output by making some simplifying assumptions. The first formula is derived from the distribution of output conditional on the coefficients, the second from the distribution of output conditional on the inputs. These approximate formulae are valid if the moments of output exist. We provide an eigenvalue condition which can be used to investigate which origo moments exist. It involves the covariance matrix of the random coefficients, the covariance matrix of the log-inputs and the order of the moments. In the empirical application we investigate, for each year in the data period, this condition, using the Maximum Likelihood (ML) estimate of the covariance matrix of the random coefficients obtained from all available data and the cross-section estimate of the covariance matrix of the log-input variables. Generally, we find that only the first and second-order origo moments of output exist. Using the approximate formulae, we provide decompositions of expected output. In order to assess the quality of the approximation formulae, a simulation experiment is performed by sampling from the two first origo moments conditional on the log-inputs. Two conclusions are drawn. The first approximate formula seems to perform better than second one for both moments, and using either formulae the approximation seems to be better for the first than for the second-order origo moment.

From both approximation formulae we derive analytical expressions for the industry production function in terms of expectations of inputs and output. The main focus in the empirical part of the paper is to estimate correct input and scale elasticities based on these expressions and compare them with those obtained when performing aggregation by analogy. However, as it is not obvious how one should define elasticities in our setting, we provide formulae for two limiting cases, denoted as variance preserving and mean preserving elasticities, respectively. While the elasticities based on analogy, by construction, are time invariant, the correct elasticities are allowed to change over time. For some inputs we find a clear trending pattern which cannot be captured by the aggregation by analogy approach. Besides, even if the variation over time is modest there are substantial level differences between the elasticities calculated from the correct formulae and those obtained by analogy, and the ranking of the inputs according to the size of the elasticities differs.

The rest of the paper is organized as follows. The model is presented in Section 2 and the properties of the distribution of output and log-output are discussed. In Section 3, we establish approximation formulae which allow the origo moments of output to be expressed be means of the expected inputs and the model's parameters. We also outline a procedure for calculating the expectation of output by simulation. Based on the analytical result in Section 3 we obtain, in Section 4, approximate aggregate production functions and derive expressions for the correct input and output elasticities according to different definitions. The data and estimation procedures are described in Section 5. Empirical results are presented in Section 6. Section 7 concludes.

2 Model and output distribution

2.1 Basic assumptions

We consider an n factor Cobb-Douglas production function, expressed in log-linear form,

$$y = x\beta + u = \alpha + z\gamma + u, \tag{1}$$

where $x = (1, z)$ is an $n+1$ dimensional *row* vector (including a one for the intercept) and $\beta = (\alpha, \gamma')'$ is an $n+1$ dimensional *column* vector (including the intercept), γ denoting the $n \times 1$ vector of input elasticities. We interpret z as $\ln(Z)$, where Z is the $1 \times n$ input vector, and y as $\ln(Y)$, where Y is output, and assume that the log-input vector, the coefficient vector, and the disturbance are independent and normally distributed:

$$x \sim \mathcal{N}(\mu_x, \Sigma_{xx}) = \mathcal{N}\left([1 \ \mu_z], \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_{zz} \end{bmatrix}\right), \tag{2}$$

$$\beta \sim \mathcal{N}(\mu_\beta, \Sigma_{\beta\beta}) = \mathcal{N}\left(\begin{bmatrix} \mu_\alpha \\ \mu_\gamma \end{bmatrix}, \begin{bmatrix} \sigma_{\alpha\alpha} & \sigma'_{\gamma\alpha} \\ \sigma_{\gamma\alpha} & \Sigma_{\gamma\gamma} \end{bmatrix}\right), \tag{3}$$

$$u \sim \mathcal{N}(0, \sigma^2), \tag{4}$$

$$x, \beta, u \;\; \text{are stochastically independent.} \tag{5}$$

The covariance matrix Σ_{xx} is singular since x has a one element, while the submatrix Σ_{zz} is non-singular in general. The covariance matrix $\Sigma_{\beta\beta}$ is also assumed to be non-singular. An implication of normality is that both β and z have infinite supports.

2.2 The distribution of log-output

We first characterize the joint distribution of the log-output, the log-input vector, and the coefficient vector. From (1), (4), and (5) it follows that

$$(y|x,\beta) \sim \mathcal{N}(x\beta, \sigma^2), \tag{6}$$

and since (1) – (5) imply $\text{var}(x\beta|\beta) = \beta'\Sigma_{xx}\beta$, $\text{var}(x\beta|x) = x\Sigma_{\beta\beta}x'$, and hence

$$\text{var}(y|\beta) = \beta'\Sigma_{xx}\beta + \sigma^2 = \text{tr}(\beta\beta'\Sigma_{xx}) + \sigma^2,$$
$$\text{var}(y|x) = x\Sigma_{\beta\beta}x' + \sigma^2 = \text{tr}(x'x\Sigma_{\beta\beta}) + \sigma^2,$$

the distribution of log-output conditional on the coefficient vector and on the log-input vector are, respectively,

$$(y|\beta) \sim \mathcal{N}(\mu_x\beta, \beta'\Sigma_{xx}\beta + \sigma^2), \tag{7}$$

$$(y|x) \sim \mathcal{N}(x\mu_\beta, x\Sigma_{\beta\beta}x' + \sigma^2). \tag{8}$$

Using the law of iterated expectations, we find

$$\mathsf{E}(y) = \mathsf{E}[\mathsf{E}(y|x)] = \mu_x\mu_\beta = \mu_y, \tag{9}$$

$$\begin{aligned} \text{var}(y) &= \mathsf{E}[\text{var}(y|\beta)] + \text{var}[\mathsf{E}(y|\beta)] = \mathsf{E}[\text{tr}(\beta\beta'\Sigma_{xx}) + \sigma^2] + \text{var}(\mu_x\beta) \\ &= \text{tr}[\mathsf{E}(\beta\beta'\Sigma_{xx})] + \sigma^2 + \mu_x\Sigma_{\beta\beta}\mu'_x \\ &= \text{tr}[(\mu_\beta\mu'_\beta + \Sigma_{\beta\beta})\Sigma_{xx}] + \sigma^2 + \mu_x\Sigma_{\beta\beta}\mu'_x \\ &= \mu_x\Sigma_{\beta\beta}\mu'_x + \mu'_\beta\Sigma_{xx}\mu_\beta + \text{tr}(\Sigma_{\beta\beta}\Sigma_{xx}) + \sigma^2 \;\; = \;\; \sigma_{yy}. \end{aligned} \tag{10}$$

The four components of σ_{yy} represent: (i) the variation in the log-inputs ($\mu_\beta' \Sigma_{xx} \mu_\beta$), (ii) the variation in the coefficients ($\mu_x \Sigma_{\beta\beta} \mu_x'$), (iii) the interaction between the variation in the log-inputs and the coefficients [$\mathrm{tr}(\Sigma_{\beta\beta}\Sigma_{xx})$], and (iv) the disturbance variation (σ^2).

2.3 The distribution of output

We next characterize the distribution of output, Y, by its origo moments. Since $Y = e^y = e^{x\beta+u}$, we know from (6) – (8) that $(Y|x,\beta)$, $(Y|x)$ and $(Y|\beta)$ follow log-normal distributions. From the normality of $(y|x,\beta)$ it follows, by using (6) and Evans, Hastings, and Peacock (1993, chapter 25), that

$$\mathsf{E}(Y^r|x,\beta) = \mathsf{E}(e^{ry}|x,\beta) = \exp[rx\beta + \tfrac{1}{2}r^2\sigma^2]. \tag{11}$$

In a similar way, (7) and (8) imply

$$\mathsf{E}(Y^r|\beta) = \mathsf{E}_{x,u}(e^{ry}|\beta) = \exp[r\mu_x\beta + \tfrac{1}{2}r^2(\beta'\Sigma_{xx}\beta + \sigma^2)], \tag{12}$$

$$\mathsf{E}(Y^r|x) = \mathsf{E}_{\beta,u}(e^{ry}|x) = \exp[rx\mu_\beta + \tfrac{1}{2}r^2(x\Sigma_{\beta\beta}x' + \sigma^2)]. \tag{13}$$

Marginally, however, Y is not log-normal, since $x\beta$ is non-normal. From (12) or (13) and the law of iterated expectations, we find that the marginal r'th order origo moment of Y can be written alternatively as

$$\mathsf{E}(Y^r) = \mathsf{E}_\beta[\mathsf{E}_{x,u}(e^{ry}|\beta)] = e^{\frac{1}{2}r^2\sigma^2}\mathsf{E}_\beta[\exp(r\mu_x\beta + \tfrac{1}{2}r^2\beta'\Sigma_{xx}\beta)], \tag{14}$$

$$\mathsf{E}(Y^r) = \mathsf{E}_x[\mathsf{E}_{\beta,u}(e^{ry}|x)] = e^{\frac{1}{2}r^2\sigma^2}\mathsf{E}_x[\exp(rx\mu_\beta + \tfrac{1}{2}r^2x\Sigma_{\beta\beta}x')]. \tag{15}$$

Using (14), and inserting for the density function of β, we have

$$\begin{aligned} \mathsf{E}(Y^r) &= \exp(\tfrac{1}{2}r^2\sigma^2)\int_{R^{n+1}} \exp[r\mu_x\beta + \tfrac{1}{2}r^2\beta'\Sigma_{xx}\beta] \\ &\qquad \times (2\pi)^{-\frac{n+1}{2}}|\Sigma_{\beta\beta}|^{-\frac{1}{2}}\exp[-\tfrac{1}{2}(\beta-\mu_\beta)'\Sigma_{\beta\beta}^{-1}(\beta-\mu_\beta)]d\beta \\ &= \exp(\tfrac{1}{2}r^2\sigma^2)(2\pi)^{-\frac{n+1}{2}}|\Sigma_{\beta\beta}|^{-\frac{1}{2}}\int_{R^{n+1}} e^{\lambda_{\beta r}}d\beta, \end{aligned} \tag{16}$$

where

$$\lambda_{\beta r} = -\tfrac{1}{2}[(\beta-\mu_\beta)'\Sigma_{\beta\beta}^{-1}(\beta-\mu_\beta) - 2r\mu_x\beta - r^2\beta'\Sigma_{xx}\beta]. \tag{17}$$

Using (15), and inserting for the density function of z, we have

$$
\begin{aligned}
(18)\quad \mathsf{E}(Y^r) &= \exp(\tfrac{1}{2}r^2\sigma^2)\int_{R^n}\exp[r(\mu_\alpha+z\mu_\gamma)+\tfrac{1}{2}r^2(\sigma_{\alpha\alpha}+2z\sigma_{\gamma\alpha}+z\Sigma_{\gamma\gamma}z')\\
&\qquad\times(2\pi)^{-\frac{n}{2}}|\Sigma_{zz}|^{-\frac{1}{2}}\exp[-\tfrac{1}{2}(z-\mu_z)\Sigma_{zz}^{-1}(z-\mu_z)']dz\\
&= \exp(r\mu_\alpha+\tfrac{1}{2}r^2(\sigma_{\alpha\alpha}+\sigma^2)(2\pi)^{-\frac{n}{2}}|\Sigma_{zz}|^{-\frac{1}{2}}\\
&\qquad\times\int_{R^n}\exp[-\tfrac{1}{2}((z-\mu_z)\Sigma_{zz}^{-1}(z-\mu_z)'-2r\mu_\gamma'z'-r^2(2z\sigma_{\gamma\alpha}+z\Sigma_{\gamma\gamma}z'))]dz\\
&= \exp(r\mu_\alpha+\tfrac{1}{2}r^2(\sigma_{\alpha\alpha}+\sigma^2)(2\pi)^{-\frac{n}{2}}|\Sigma_{zz}|^{-\frac{1}{2}}\int_{R^n}e^{\lambda_{zr}}dz,
\end{aligned}
$$

where

$$
(19)\qquad \lambda_{zr}=-\tfrac{1}{2}[(z-\mu_z)\Sigma_{zz}^{-1}(z-\mu_z)'-2r\mu_\gamma'z'-r^2(2z\sigma_{\gamma\alpha}+z\Sigma_{\gamma\gamma}z')].
$$

Eqs. (16) and (18) show that in order to evaluate $\mathsf{E}(Y^r)$ exactly, we have to evaluate either of the multiple integrals $\int_{R^{n+1}}e^{\lambda_{\beta r}}d\beta$ and $\int_{R^n}e^{\lambda_{zr}}dz$, whose integrands are both exponential functions with one linear term and two quadratic forms in the exponent. We show in Appendix A that

$$
(20)\qquad \left\{\begin{array}{c}
\displaystyle\int_{R^{n+1}}e^{\lambda_{\beta r}}d\beta \;\text{ and }\; \int_{R^n}e^{\lambda_{zr}}dz \;\text{ exist}\\
\Longleftrightarrow\\
\text{all eigenvalues of }\; \Sigma_{\beta\beta}^{-1}-r^2\Sigma_{xx}\;\text{ are strictly positive.}
\end{array}\right.
$$

A condition of this kind is a consequence of assuming that β and z have both infinite supports.

3 Approximations to the origo moments of output

We now present two ways of obtaining *approximate* closed form expressions for $\mathsf{E}(Y^r)$, one based on (14) and one based on (15). To check the numerical accuracy, we also describe a way of computing numerical approximations to (15).

3.1 Analytical approximations

We first let $\delta=\beta-\mu_\beta\sim\mathcal{N}(0,\Sigma_{\beta\beta})$ and rewrite (14) as

$$
\begin{aligned}
(21)\qquad \mathsf{E}(Y^r) &= e^{\frac{1}{2}r^2\sigma^2}\mathsf{E}\left[\exp[r\mu_x\mu_\beta+r\mu_x\delta+\tfrac{1}{2}r^2\mu_\beta'\Sigma_{xx}\mu_\beta+r^2\mu_\beta'\Sigma_{xx}\delta+\tfrac{1}{2}r^2\delta'\Sigma_{xx}\delta]\right]\\
&= \exp\left[r\mu_x\mu_\beta+\tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta+\sigma^2)\right]\mathsf{E}\left[\exp[(r\mu_x+r^2\mu_\beta'\Sigma_{xx})\delta+\tfrac{1}{2}r^2\delta'\Sigma_{xx}\delta]\right].
\end{aligned}
$$

The exponent in the expression after the last expectation operator is the sum of a normally distributed variable and a quadratic form in a normally distributed vector. Since its distribution is complicated, we, for simplicity, replace $\delta'\Sigma_{xx}\delta = \mathrm{tr}[\delta\delta'\Sigma_{xx}]$ by its expectation, $\mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}]$. We then get from (21), provided that (20) holds, the following approximation to the r'th origo moment of output:

$$
\begin{aligned}
\text{(22)}\qquad \mathsf{E}(Y^r) \approx G_{\beta r}(Y) &= \exp\left[r\mu_x\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}] + \sigma^2)\right] \\
&\quad \times \exp\left[\tfrac{1}{2}(r\mu_x + r^2\mu_\beta'\Sigma_{xx})\Sigma_{\beta\beta}(r\mu_x + r^2\mu_\beta'\Sigma_{xx})'\right] \\
&= \exp\Big[r\mu_x\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \mu_x\Sigma_{\beta\beta}\mu_x' + \mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}] + \sigma^2) \\
&\qquad + r^3\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\mu_x' + \tfrac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta\Big],
\end{aligned}
$$

since $\mathrm{var}[(r\mu_x + r^2\mu_\beta'\Sigma_{xx})\delta] = (r\mu_x + r^2\mu_\beta'\Sigma_{xx})\Sigma_{\beta\beta}(r\mu_x + r^2\mu_\beta'\Sigma_{xx})'$.

We next let $v = x - \mu_x \sim \mathcal{N}(0, \Sigma_{xx})$ and rewrite (15) as

$$
\begin{aligned}
\text{(23)}\qquad \mathsf{E}(Y^r) &= e^{\frac{1}{2}r^2\sigma^2}\,\mathsf{E}\left[\exp[r\mu_x\mu_\beta + rv\mu_\beta + \tfrac{1}{2}r^2\mu_x\Sigma_{\beta\beta}\mu_x' + r^2v\Sigma_{\beta\beta}\mu_x' + \tfrac{1}{2}r^2v\Sigma_{\beta\beta}v']\right] \\
&= \exp\left[r\mu_x\mu_\beta + \tfrac{1}{2}r^2(\mu_x\Sigma_{\beta\beta}\mu_x' + \sigma^2)\right]\mathsf{E}\left[\exp[(r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})v' + \tfrac{1}{2}r^2v\Sigma_{\beta\beta}v']\right].
\end{aligned}
$$

Again, the exponent in the expression after the last expectation operator is the sum of a normally distributed variable and a quadratic form in a normally distributed vector. We, for simplicity, replace $v\Sigma_{\beta\beta}v' = \mathrm{tr}[v'v\Sigma_{\beta\beta}]$ by its expectation, $\mathrm{tr}[\Sigma_{xx}\Sigma_{\beta\beta}]$, and get from (23), provided that (20) holds, the following alternative approximation to the r'th order origo moment of output:

$$
\begin{aligned}
\text{(24)}\qquad \mathsf{E}(Y^r) \approx G_{xr}(Y) &= \exp\left[r\mu_x\mu_\beta + \tfrac{1}{2}r^2(\mu_x\Sigma_{\beta\beta}\mu_x' + \mathrm{tr}[\Sigma_{xx}\Sigma_{\beta\beta}] + \sigma^2)\right] \\
&\quad \times \exp\left[\tfrac{1}{2}(r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})\Sigma_{xx}(r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})'\right] \\
&= \exp\Big[r\mu_x\mu_\beta + \tfrac{1}{2}r^2(\mu_x\Sigma_{\beta\beta}\mu_x' + \mu_\beta'\Sigma_{xx}\mu_\beta + \mathrm{tr}[\Sigma_{xx}\Sigma_{\beta\beta}] + \sigma^2) \\
&\qquad + r^3\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\mu_x' + \tfrac{1}{2}r^4\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu_x'\Big],
\end{aligned}
$$

since $\mathrm{var}[(r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})v'] = (r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})\Sigma_{xx}(r\mu_\beta' + r^2\mu_x\Sigma_{\beta\beta})'$.

The expressions after the last equality sign in (22) and (24) coincide, except for the last term in the exponents. This term is $\frac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta$ when using the approximation derived from the expectation conditional on β, *i.e.*, (14), and the symmetric expression $\frac{1}{2}r^4\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu'_x$ when using the approximation derived from the expectation conditional on x, *i.e.*, (15). We can then write the two approximations to $\mathsf{E}(Y^r)$ as

$$G_{\beta r}(Y) = \Phi_r(y)\Gamma_r\Lambda_{\beta r}, \qquad G_{xr}(Y) = \Phi_r(y)\Gamma_r\Lambda_{xr}, \tag{25}$$

where

$$\Phi_r(y) = \exp\left[r\mu_y + \frac{1}{2}r^2\sigma_{yy}\right], \tag{26}$$

$$\Gamma_r = \exp\left[r^3\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta\right], \tag{27}$$

$$\Lambda_{\beta r} = \exp\left[\frac{1}{2}r^4\mu'_\beta\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta\right], \qquad \Lambda_{xr} = \exp\left[\frac{1}{2}r^4\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu'_x\right]. \tag{28}$$

The first term in (25), $\Phi_r(y)$, is the approximation we would have obtained if we had proceeded as if y were normally and Y were log-normally distributed marginally [cf. (9) and (10)], and hence it may be viewed as a kind of 'first-order' approximation. The second and third terms, Γ_r, $\Lambda_{\beta r}$ and Λ_{xr}, where $\Lambda_{\beta r}$ is used if we rely on (22) and Λ_{xr} is used if we rely on (24), are correction factors to this first-order approximation.

3.2 Numerical approximations

There are several methods for approximating the moments numerically. One is to evaluate the multivariate integrals in (16) or (18) using quadrature methods [see, *e.g.*, Greene (2003, Appendix E.5.4)]. A simpler and more robust method, albeit computationally more intensive, is to simulate the expectations in (14) or (15). The idea is simple and well known: estimating the expectation in a distribution by a corresponding sample average based on synthetic data.

To obtain this we first define the variables $V(x;r) = \exp(rx\mu_\beta + \frac{1}{2}r^2x\Sigma_{\beta\beta}x')$, $r = 1,2,\dots$. Next, we draw a sample of x's from the $\mathcal{N}(\mu_x, \Sigma_{xx})$ distribution[2] and, for each element in the sample, calculate $V(x;r)$. Finally, the sample averages of these V's are used as estimators for the corresponding expectations, the $\mathsf{E}[V(x;r)]$'s. As long as the r'th origo moment of Y exists, cf. (20), the law of large numbers ensures that the sample average converges in probability towards the expectation.

[2]The random number generator *g05ezf* in NAG's library of *Fortran77* routines (Mark 16) was used.

4 An approximate aggregate production function in origo moments

We now derive approximate relationships between $\mathsf{E}(Y^r)$ and $\mathsf{E}(Z^r)$ to be used in examining aggregation biases in the production function parameters when the aggregate variables are represented by their arithmetic means. In doing this, we note that $e^{\mathsf{E}[\ln(Y)]}$ and $e^{\mathsf{E}[\ln(Z_i)]}$ can be associated with the geometric means, and $\mathsf{E}(Y)$ and $\mathsf{E}(Z_i)$ with the arithmetic means of the output and the i'th input, respectively. We initially consider an arbitrary value of r, assuming that (20) is satisfied, and then discuss the case $r=1$ in more detail.

4.1 An aggregate Cobb-Douglas production function

Let

$$
\begin{aligned}
\theta_{y\beta r} &= \ln[G_{\beta r}(Y)] - r\mu_y = \ln[\Phi_r(y)\Gamma_r\Lambda_{\beta r}] - r\mu_x\mu_\beta \\
&= \tfrac{1}{2}r^2\sigma_{yy} + r^3\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta, \\
\theta_{yxr} &= \ln[G_{xr}(Y)] - r\mu_y = \ln[\Phi_r(y)\Gamma_r\Lambda_{xr}] - r\mu_x\mu_\beta \\
&= \tfrac{1}{2}r^2\sigma_{yy} + r^3\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}r^4\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu_x',
\end{aligned} \tag{29}
$$

which can be interpreted as two alternative approximations to $\ln[\mathsf{E}(Y^r)] - \mathsf{E}[\ln(Y^r)]$. Further, let Z_i denote the i'th element of Z, *i.e.*, the i'th input, and $z_i = \ln(Z_i)$. From (2) it follows that

$$z_i \sim \mathcal{N}(\mu_{zi}, \sigma_{zizi}), \qquad i = 1, \ldots, n,$$

where μ_{zi} is the i'th element of μ_z and σ_{zizi} is the i'th diagonal element of Σ_{zz}. Hence,

$$\mathsf{E}(Z_i^r) = \mathsf{E}(e^{z_i r}) = \exp\left(\mu_{zi}r + \tfrac{1}{2}\sigma_{zizi}r^2\right), \qquad r = 1, 2, \ldots\ ;\ i = 1, \ldots, n. \tag{30}$$

Let $\mu_{\gamma i}$ be the i'th element of μ_γ, *i.e.*, the expected input elasticity of the i'th input. Since (30) implies $e^{\mu_{zi}\mu_{\gamma i}r} = \exp(-\frac{1}{2}\sigma_{zizi}r^2\mu_{\gamma i})[\mathsf{E}(Z_i^r)]^{\mu_{\gamma i}}$, it follows from (22) and (24) that

$$
\begin{aligned}
G_{\beta r}(Y) &= e^{\mu_\alpha r} A_{\beta r} \prod_{i=1}^n [\mathsf{E}(Z_i^r)]^{\mu_{\gamma i}}, \\
G_{xr}(Y) &= e^{\mu_\alpha r} A_{xr} \prod_{i=1}^n [\mathsf{E}(Z_i^r)]^{\mu_{\gamma i}},
\end{aligned} \tag{31}
$$

where

$$A_{\beta r} = \exp\left(\theta_{y\beta r} - \tfrac{1}{2}r^2\textstyle\sum_{i=1}^{n}\sigma_{zizi}\mu_{\gamma i}\right) = \exp(\theta_{y\beta r} - \tfrac{1}{2}r^2\mu_\gamma'\sigma_{zz}),$$
$$A_{xr} = \exp\left(\theta_{yxr} - \tfrac{1}{2}r^2\textstyle\sum_{i=1}^{n}\sigma_{zizi}\mu_{\gamma i}\right) = \exp(\theta_{yxr} - \tfrac{1}{2}r^2\mu_\gamma'\sigma_{zz}), \tag{32}$$

and $\sigma_{zz} = \text{diagv}(\Sigma_{zz})$.[3] Eq. (31) can be interpreted (approximately) as a *Cobb-Douglas function in the r'th origo moments of* Y *and* $Z_1,\dots,Z_n$ with exponents equal to the expected micro elasticities $\mu_{\gamma 1},\dots,\mu_{\gamma n}$ and an intercept $e^{\mu_\alpha r}$, adjusted by either of the factors $A_{\beta r}$ or A_{xr}. These factors depend, via $\theta_{y\beta r}$ and θ_{yxr} [cf. (9), (10) and (29)], on the first and second moments of the log-input vector x, the coefficient vector β, and the disturbance u. For $r = 1$, (31) gives in particular

$$G_{\beta 1}(Y) = e^{\mu_\alpha} A_{\beta 1} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}},$$
$$G_{x1}(Y) = e^{\mu_\alpha} A_{x1} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}. \tag{33}$$

At a first glance, it seems that this equation could be interpreted as a Cobb-Douglas function in the arithmetic means $\mathsf{E}(Y)$ and $\mathsf{E}(Z_1),\dots,\mathsf{E}(Z_n)$, with elasticities coinciding with the expected micro elasticities $\mu_{\gamma 1},\dots,\mu_{\gamma n}$ and an intercept e^{μ_α} adjusted by the factor $A_{\beta 1}$ or A_{x1}. However, we will show below that the situation is not so simple.

4.2 Aggregation by analogy and aggregation biases in output and in input elasticities

Assume now that we, instead of (33), use as our aggregate production function the function obtained by *aggregating by analogy* from arithmetic means, *i.e.*,

$$\widehat{\mathsf{E}(Y)} = e^{\mu_\alpha} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}. \tag{34}$$

This can be said to mimic the aggregation by analogy often used by macro-economists and macro model builders. The resulting *aggregation error in output* when we use the approximate formula for $\mathsf{E}(Y)$ is

$$\epsilon_\beta(Y) = G_{\beta 1}(Y) - \widehat{\mathsf{E}(Y)} = (A_{\beta 1} - 1)e^{\mu_\alpha} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}},$$
$$\epsilon_x(Y) = G_{x1}(Y) - \widehat{\mathsf{E}(Y)} = (A_{x1} - 1)e^{\mu_\alpha} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}. \tag{35}$$

[3] We here and in the following use 'diagv' to denote the column vector containing the diagonal elements of the following square matrix.

We next consider the aggregate input elasticities and their biases, still representing the exact parametric aggregate production function by its approximation (33) and the incorrect one by (34). The latter way of aggregating the Cobb-Douglas production function will bias not only its intercept, but also its derived input elasticities, because $A_{\beta 1}$ and A_{x1} respond to changes in μ_z and Σ_{zz}. From (9), (10) and (29) we see that *when* $\Sigma_{\gamma\gamma}$ *is non-zero, a change in* μ_z *affects not only the expectation of* $\ln(Y)$, but also its variance σ_{yy}, as well as $\theta_{y\beta 1}$ and θ_{yx1}. Eqs. (9), (10), (22) and (24) imply

$$\tag{36} \begin{aligned} \ln[G_{\beta 1}(Y)] &= \mu_y + \tfrac{1}{2}\sigma_{yy} + \mu_x \Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}\mu'_\beta \Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta, \\ \ln[G_{x1}(Y)] &= \mu_y + \tfrac{1}{2}\sigma_{yy} + \mu_x \Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}\mu_x \Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu'_x. \end{aligned}$$

Using the fact that, from (30), $\Delta \ln[\mathsf{E}(Z)]' = \Delta(\mu'_z + \frac{1}{2}\sigma_{zz})$, we show in Appendix B that

$$\tag{37} \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mu^*_{\gamma\beta} = \mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma \quad \text{when } \Sigma_{zz} \text{ is constant}, \\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mu^*_{\gamma x} = (I + \Sigma_{\gamma\gamma}\Sigma_{zz})(\mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z) \quad \text{when } \Sigma_{zz} \text{ is constant}, \end{aligned}$$

$$\tag{38} \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mu^{**}_{\gamma\beta} = \operatorname{diagv}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma} + 2\mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \mu_\gamma\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu'_\gamma) \\ &\quad \text{when } \mu_z \text{ and the off-diagonal elements of } \Sigma_{zz} \text{ are constant}, \\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mu^{**}_{\gamma x} = \operatorname{diagv}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma} + 2\mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\mu'_z\mu_z\Sigma_{\gamma\gamma}) \\ &\quad \text{when } \mu_z \text{ and the off-diagonal elements of } \Sigma_{zz} \text{ are constant}. \end{aligned}$$

From these formulae it is not obvious how we should define and measure the exact aggregate input elasticity of input i, interpreted as $(\partial \ln[\mathsf{E}(Y)])/(\partial \ln[\mathsf{E}(Z_i)])$, since, in general, both the mean and the variance vector of the *log-input* distribution change over time. Eq. (37) may be interpreted as a vector of *dispersion preserving* aggregate input elasticities, and eq. (38) as a vector of *mean preserving* elasticities. Anyway, μ_γ provides a biased measure of the aggregate elasticity vector. The bias vector implied by the dispersion preserving macro input elasticities, obtained from (37), is

$$\tag{39} \begin{aligned} \epsilon_\beta(\mu_\gamma) &= \mu^*_{\gamma\beta} - \mu_\gamma = \Sigma_{\gamma\gamma}\left(\mu'_z + \Sigma_{zz}\mu_\gamma\right), \\ \epsilon_x(\mu_\gamma) &= \mu^*_{\gamma x} - \mu_\gamma = \Sigma_{\gamma\gamma}\left(\mu'_z + \Sigma_{zz}\mu_\gamma + \Sigma_{zz}\Sigma_{\gamma\gamma}\mu'_z\right). \end{aligned}$$

The bias vectors for the mean preserving elasticities can be obtained from (38) in a similar way.

The dispersion preserving elasticities may be of most interest in practice, since constancy of the *variance* of the log of input i, *i.e.*, σ_{zizi}, implies constancy of the *coefficient* *of variation* of the untransformed input i. This will follow when the i'th input of all micro units change proportionally.[4] This is seen from the following expression for the coefficient of variation of Z_i [cf. (30) and Evans, Hastings, and Peacock (1993, chapter 25)]:

$$v(Z_i) = \frac{\text{std}(Z_i)}{\mathsf{E}(Z_i)} = (e^{\sigma_{zizi}} - 1)^{\frac{1}{2}}\,. \tag{40}$$

5 Econometric model, data and estimation

We next turn to the parametrization of the micro production (1), the data, and the estimation procedure. We specify four inputs: ($n = 4$), capital (K), labour (L), energy (E) and materials (M), and include a deterministic linear trend (t), intended to capture the level of the technology. We parametrize (1) as

$$y_{it} = \alpha_i^* + \kappa t + \textstyle\sum_j \beta_{ji} x_{jit} + u_{it}, \qquad j,k = K,L,E,M, \tag{41}$$

where subscripts i and t denote plant and year of observation, respectively, $y_{it} = \ln(Y_{it})$, $x_{jit} = \ln(X_{jit})$ $(j = K,L,E,M)$, and α_i^* and β_{ji} $(j = K,L,E,M)$ are *random coefficients* specific to plant i, and κ is plant invariant. The disturbance $u_{it} \sim \mathcal{N}(0, \sigma_{uu})$. We let $x_{it} = (x_{Kit}, x_{Lit}, x_{Eit}, x_{Mit})'$, collect all the random coefficients for plant i in the vector $\psi_i = (\alpha_i^*, \beta_{Ki}, \beta_{Li}, \beta_{Ei}, \beta_{Mi})'$, and describe the heterogeneity in the model structure as follows: All x_{it}, u_{it}, and ψ_i are independently distributed, with

$$\mathsf{E}(\psi_i) = \psi = (\bar{\alpha}^*, \bar{\beta}_K, \bar{\beta}_L, \bar{\beta}_E, \bar{\beta}_M)', \quad \mathsf{E}[(\psi_i - \psi)(\psi_i - \psi)'] = \Omega,$$

where Ω is a symmetric, but otherwise unrestricted matrix.

Since our focus will be on aggregation biases on a yearly basis it is convenient to rewrite (41) as

$$y_{it} = \alpha_{it} + \textstyle\sum_j \beta_{ji} x_{jit} + u_{it}, \qquad j = K,L,E,M, \tag{42}$$

where $\alpha_{it} = \alpha_i^* + \kappa t$, satisfying $E(\alpha_{it}) = \bar{\alpha}_t = \bar{\alpha}^* + \kappa t$. In the following we sometimes suppress the indices for plant and year and write (42) as (1) with $j,k = K,L,E,M$.

[4]The mean preserving elasticities relate to a more 'artificial' experiment in which $\mathsf{E}[\ln(Z_i)]$ is kept fixed and $v(Z_i)$ is increased, *i.e.*, $\text{std}(Z_i)$ is increased relatively more than $\mathsf{E}(Z_i)$.

The unknown parameters are estimated by ML, using the PROC MIXED procedure in the SAS/STAT software [see Littell *et al.* (1996)]. Positive definiteness of Ω is imposed as an *a priori* restriction. This application draws on the estimation results in Biørn, Lindquist and Skjerpen (2002, in particular Section 2 and Appendix A). The data are unbalanced panel data for the years 1972 – 1993 from two Norwegian manufacturing industries, *Pulp and paper* and *Basic metals*. A further description is given in Appendix C. The estimates, as well as the estimates of the mean scale elasticity $\bar{\beta} = \sum_j \bar{\beta}_j$, are given in Appendix E.

6 Empirical results

6.1 Tests of the normality of the log-input distribution

Since this study relies on log-normality of the inputs, we present, in Appendix D, the results of univariate statistical tests of whether, for each year in the sample period, log-output and log-inputs are normally distributed. The test statistic takes into account both skewness and excess kurtosis. Summary results are presented in Table 1. Log-normality is in most cases not rejected. However, for Pulp and paper, some evidence of non-normality, especially in the first years in the sample, is found. Non-normality is most pronounced for energy and materials, and normality is rejected at the 1 per cent significance level for both of these inputs in the first five years. Despite these irregularities, we conclude from these results that (2) is an acceptable simplifying assumption for the study.

6.2 Simulations of the origo moments of output

Before embarking on the task of simulating the origo moments of output, one should check whether or not the condition for their existence, (20), is met. We found that for both industries, the first and second-order moments exist in all years, except for Basic metals in 1993 where only the first-order moment exists. For Pulp and paper also the third-order moment exists in 1992.

The fact that the highest existing moments are of low order may cause problems that should not be neglected. Estimates of moments based on simulated sample averages are of little value unless accompanied by measures of the sampling error, such as confidence intervals. However, in order to obtain confidence intervals one usually relies on standard central limit theorems, thereby assuming the existence of both the expectation and the variance of the random variable in question. If we let $\bar{r} \geq 1$ denote the highest existing moment and regard $Y^{\bar{r}}$ as a random variable, it is clear that $\text{var}(Y^{\bar{r}}) = \mathsf{E}(Y^{2\bar{r}}) - [\mathsf{E}(Y^{\bar{r}})]^2$ does not exist, since $\mathsf{E}(Y^{2\bar{r}})$ does not exist by definition.

In this case, a generalization of the central limit theorem is appropriate, see McCulloch (1986) and Embrechts *et al.* (1997, pp. 71–81) for the points to follow. In general, the distribution of a sample average of n IID random variables converges towards a stable distribution characterized by four parameters and denoted by $S(\alpha, \beta, c, \delta)$, where α is the characteristic exponent, β is a skewness parameter, while c and δ determines scale and location, respectively. The shape of the distribution is determined by α and β, while c compresses or extends the distribution about δ. The standard central limit theorem is a special case: if both the expectation and the variance of the IID variables exist, $\alpha = 2$ and the limiting distribution is the normal. If the expectation, but not the variance, of the IID variables exists, $1 < \alpha < 2$. Several familiar features of the normal distribution are also generally valid for stable distributions, one of them is invariance under averaging.[5] The crux of the problem of simulating the highest existing origo moment is the following: consistency of the sample average of output as an estimator of its expectation is ensured as long as its theoretical moment exists, but inaccuracy in the estimate may be persistent even for very large samples.

We have simulated the first and second-order moments of output, using 10^8 synthetic observations for every year in each of the two industries. Each of the samples of 10^8 observations have been divided in 10^4 sub-samples, and sample averages for the sub-samples have been calculated, enabling us to study the distribution of the sub-sample averages. Provided that these distributions belong to the stable class, estimated distribution parameters will be applicable to the total sample since the total average is the average of

[5] More precisely, if n IID random variables are drawn from a stable distribution $S(\alpha, \beta, c, \delta)$, their average will also have a stable distribution $S(\alpha, \beta, cn^{(1/\alpha)-1}, \delta)$, cf. McCulloch (1986, pp. 1122-1123). In the normal case, with $\alpha = 2$, the scale parameter of the average equals $cn^{-1/2}$. This implies that the distribution of the average is more compressed than the original distribution, and thus the width of confidence intervals will be rapidly decreasing in n. In the case where α is close to 1, the factor $n^{(1/\alpha)-1}$ is close to 1, implying that the width of confidence intervals decreases slowly.

all sub-sample averages, due to the invariance under averaging property.[6]

Parameters in stable distributions can, in principle, be estimated by maximum likelihood, but this is rather difficult. McCulloch (1986) suggest a far simpler, albeit less efficient, method based on functions of sample quantiles. Using this latter method, we found for both industries estimates of α in the interval (0.7, 0.9). Typical estimates of β were above 0.75, indicating strong right skewness.

A full treatment of this subject is beyond our scope. For the second-order moments, we simply report the average, the 5 per cent, and the 95 per cent quantile in the distribution of sub-sample averages in Table 2b. The average exceeds the 95 per cent quantile in almost every year, due to the heavy upper tail. First-order moments are reported, with normal confidence intervals, in Table 2a.

6.3 Decompositions of the origo moments of output

Tables 3 – 8 present the decomposition of the log of expected output for Pulp and paper and Basic metals. Tables 3 and 4 give, respectively, the decomposition of the log of expected output and the log of the second-order moment of output according to the first formulae in (25). The corresponding results based on the second formula are given in Tables 5 and 6. In Tables 7 and 8 we report on a further decomposition of the factor $\ln[\Psi_r(y)]$ $(r = 1, 2)$, which is common to both decomposition formulae. In Table 3 we decompose the log of expected output in three parts. We also compare the estimate of the log of expected output with the corresponding results based on simulations as outlined earlier. The first column for both industries gives the log of expected output if one proceeds as if output were log-normally distributed, which is not in accordance with our stochastic assumptions. In Table 7 we perform a further decomposition of $\ln[\Psi_1(y)]$, into five sub-components. The first column for each industry in Table 7 shows the downward bias caused by the naive way of representing the expectation of a log-normal variable, say Z, by $e^{\mathsf{E}[\ln(Z)]}$. We note that the results based on the approximation formulae (22) agree more closely with the simulation results than those based on the alternative formula (24). This is most pronounced for the log of the second-order origo moment.

[6]Note that this is a simplifying assumption, and that there is no guarantee that the distribution of sub-sample averages is stable even when each sub-sample consists of 10^4 observations.

We observe from Table 3 that taking account of the correction factors $\ln(\Gamma_1)$ and $\ln(\Lambda_{\beta 1})$, generally reduces the discrepancy between results based on the approximate analytical formulae and the simulation results. This is true for both industries, except for Basic metals in the ultimate year. Note also that $\ln(\Gamma_1)$ yields a negative and $\ln(\Lambda_{\beta 1})$ a positive contribution. The absolute value of the latter is, however, generally larger than the former. For the (log of the) second-order moment, Table 4 reveals that there is a positive discrepancy between the simulation results and the log of the second-order origo moment calculated from the approximation formula (22). Thus, the approximation formula seems to perform better for the log of first-order than for the log of the second-order moment. This may be due to the fact that condition (20) is closer to being violated for $r = 2$ than for $r = 1$; cf. Section 6.2.

Using the approximation formulae (24), we see that the the total effect of including the two correction terms $\ln(\Gamma_1)$ and $\ln(\Lambda_{x1})$ is to widen the gap between the results from simulations and from analytical formulae. This is the case for both industries. Besides, the absolute value of $\ln(\Lambda_{x1})$ is very small and for practical purposes negligible. From Tables 7 and 8 we see that all sub-components contribute positively. For the first-order moment the largest contribution comes from μ_y followed by the term picking up the contribution from the variation in the random coefficients. Smaller contributions are given by the variation in log-inputs, the interaction term and the term representing the variance of the genuine disturbances. For the second-order moments the effect of the random variation in coefficients contributes more than the effects from $2\mu_y$.

6.4 Scale and input elasticities

In Tables 9 – 12 we report on four types of input and scale elasticities at the industry level for Pulp and paper and Basic Metals. Tables 9 and 11 are based on the approximation formula $G_{\beta 1}$, whereas Tables 10 and 12 are based on the approximation formula G_{x1}. We label the elasticities in Tables 9 and 10 dispersion preserving macro elasticities and the elasticities in Tables 11 and 12 mean preserving macro elasticities. The companion, time invariant, micro elasticities are reported in Table E.1. We see that the micro elasticities lie between the dispersion preserving and mean preserving micro elasticities irrespective of which approximation formula is applied. The energy elasticity at the industry level mainly comes out as negative when using the dispersion preserving elasticity formulae, but they are positive in the mean preserving case. However, the micro energy elasticity is also low, especially for Pulp and paper. Many of the elasticities do not change very much over time, but there are important exceptions. Consider, for instance, the labour elasticity for Basic metals which has a negative trend over time regardless of which elasticity formula is used. At the micro level, the materials elasticity was found to be the largest among the input elasticities in all industries, whereas labour possesses this property at the industry level. Within Pulp and paper also the capital elasticity is higher than the materials elasticity at the industry level. Of course one can also calculate time-varying weighted elasticities between these two 'limiting' cases. Also these elasticities emphasize the arguments against using 'raw' micro elasticities in macro contexts. Since the macro elasticties are quite different from the micro elasticities and some of them trends over time, policy conclusions based on the micro parameters have the potential to be misleading.

7 Conclusions

In this paper, we consider aggregation of Cobb-Douglas production functions from the micro to the industry level when the production function parameters as well as the log-input variables are assumed to be multivariate normally distributed. Although output will then not be log-normally distributed marginally, we are able to provide analytical approximation formulae for both the expectation and the higher-order origo moments of the output distribution. One is derived from the conditional distribution of output given the inputs, the other from the conditional distribution of output given the parameters. We also give conditions for the existence of the origo moments. These conditions turn out to be rather strong in the present case, as only the two first origo moments of the output distribution exist. This, inter alia, seems to be due to our assumption that the distribution of the log-inputs and the coefficients are normal and hence have infinite supports. This suggests directions for future research, even if relaxation of normality will, most likely, increase the analytical and numerical problems. To evaluate the quality of the approximate formulae, we supplement the analytical formulae with simulation experiments.

We derive the industry level production function, expressed as a relationship between expected inputs and expected output, and bias formulae obtained when comparing correctly aggregated input and scale elasticities with elasticities obtained from the micro level, denoted as aggregation by analogy. However, as it is not obvious what should be meant by aggregate elasticities, we give different definitions, based on different assumptions about how the distribution of the micro variables is restricted in the aggregation process. Our modeling framework is applied to two unbalanced panel data sets for the Norwegian Pulp and paper and Basic metals industries.

We demonstrate different ways of decomposing expected log of output. One of the components is the one we get when erroneously assuming that output is log-normally distributed marginally. When additional terms are included in the approximation formula, exploiting the distribution of output conditional on the inputs, we obtain results that agree better with those obtained by the simulations. The opposite is the case when we apply the distribution with the reverse conditioning.

With respect to industry level input and scale elasticities, we present results for two limiting cases, labeled as variation preserving and mean preserving elasticities. We find the scale elasticities to be uniformly higher at the industry than at the micro level for industries. Besides, the ranking of the input elasticities by size is not the same at the micro and the industry level. Unlike the micro elasticities, which are, by assumption, time-invariant, the elasticities at the industry level change over time. For some elasticities we find a clear trending pattern over the sample period. It is thus safe to conclude that the aggregation by analogy strategy followed by many macro economists is far from innocent and may lead to wrong conclusions.

Throughout this paper, we have assumed that production function parameters and log-inputs are uncorrelated. An interesting extension would be to relax this assumption. This can, for instance, be done within a model in which all parameters are fixed and plant specific. However, this will imply that a substantial part of the sample must be wasted, since we need a minimum number of observations for each plant to estimate the plant specific parameters properly. It is not clear whether the approach pursued in this paper can be applied to more flexible functional forms, such as the CES, the Translog, or the Generalized Leontief production functions. Probably, it will be harder to obtain useful analytical approximation formulae for expected output, and since the production functions involve higher-order terms, the problems related to the non-existence of higher-order origo moments of output will most likely be aggravated. Consequently, in such cases it may be more fruitful to stick to an aggregation approach where the assumption that parameters and log-input variables are drawn from a specific parametric distributions are relaxed, as exemplified in Biørn and Skjerpen (2002).

Table 1. Testing of normality of log-transformed variables. Numbers of years (out of 22) in which the statistic is significant at the indicated significance level

	log(X)	log(K)	log(L)	log(E)	log(M)
Pulp and paper					
Skewness:					
1 per cent	1	0	0	0	5
5 per cent	7	8	2	1	10
10 per cent	9	10	4	1	12
Kurtosis:					
1 per cent	1	0	0	8	0
5 per cent	4	0	0	21	0
10 per cent	7	0	1	22	0
Normality:					
1 per cent	0	0	0	7	3
5 per cent	9	5	1	12	6
10 per cent	11	8	3	20	10
Basic metals					
Skewness					
1 per cent	0	0	0	0	0
5 per cent	1	0	0	0	0
10 per cent	4	0	0	0	0
Kurtosis					
1 per cent	0	0	0	0	0
5 per cent	8	2	9	1	2
10 per cent	14	15	18	11	10
Normality					
1 per cent	0	0	0	0	0
5 per cent	1	0	0	1	0
10 per cent	11	0	4	7	0

Table 2a. Simulated first order moments[1] with confidence intervals, in logarithms

	Moment	Lower limit	Upper limit	Moment	Lower limit	Upper limit
	Pulp and paper			Basic metals		
1972	5.9642	5.9619	5.9665	6.7566	6.7491	6.7604
1973	6.0421	6.0396	6.0445	6.8375	6.8286	6.8419
1974	6.2679	6.2633	6.2724	6.7475	6.7414	6.7507
1975	6.1488	6.1443	6.1532	6.9647	6.9540	6.9701
1976	6.1558	6.1511	6.1604	6.7192	6.7115	6.7231
1977	5.9498	5.9471	5.9524	6.7825	6.7733	6.7871
1978	5.9708	5.9681	5.9734	6.4945	6.4886	6.4976
1979	6.0949	6.0923	6.0974	6.9541	6.9461	6.9581
1980	6.1343	6.1321	6.1364	6.9318	6.9252	6.9351
1981	6.2062	6.2037	6.2086	6.8530	6.8469	6.8561
1982	6.2892	6.2863	6.2921	6.7398	6.7341	6.7428
1983	6.2855	6.2836	6.2873	6.8360	6.8300	6.8391
1984	6.2953	6.2936	6.2970	7.0430	7.0375	7.0458
1985	6.4049	6.4030	6.4067	7.0030	6.9988	7.0051
1986	6.3942	6.3923	6.3961	7.0995	7.0955	7.1016
1987	6.4191	6.4169	6.4212	7.2459	7.2403	7.2488
1988	6.4893	6.4869	6.4918	7.3868	7.3813	7.3896
1989	6.4741	6.4721	6.4762	7.3270	7.3229	7.3291
1990	6.4467	6.4445	6.4489	7.3848	7.3798	7.3874
1991	6.5224	6.5201	6.5246	7.3421	7.3375	7.3444
1992	6.2779	6.2770	6.2788	7.1605	7.1585	7.1615
1993	6.3373	6.3362	6.3384	7.2189	7.2151	7.2209

1. Moments are averages over 10^8 synthetic observations.

Table 2b. Simulated second order moments[1] and percentiles[2] in distribution of sample averages

	Pulp and paper			Basic metals		
	Moment	5% perc.	95% perc.	Moment	5% perc.	95% perc.
1972	20.3222	16.2083	19.9855	23.6420	18.6538	23.1054
1973	20.7355	16.4392	20.2932	24.3111	18.8108	23.3087
1974	23.3204	17.3739	21.9618	23.0945	18.3958	22.4790
1975	22.9392	17.0538	21.5681	25.0652	19.1299	23.8377
1976	23.0748	17.0582	21.5762	23.6211	18.3669	22.6613
1977	20.7370	16.1659	20.0623	24.0757	18.4756	22.9266
1978	20.7938	16.2440	20.1622	22.1762	17.6244	21.5623
1979	20.9894	16.5070	20.3702	23.9905	18.7571	23.0237
1980	20.5820	16.4588	20.0643	23.4015	18.6020	22.6752
1981	21.1147	16.7027	20.4655	22.9574	18.3655	22.3305
1982	21.9550	17.1010	21.1639	22.5748	18.1302	22.0074
1983	20.4060	16.6433	19.9861	23.0892	18.4202	22.3085
1984	20.2310	16.6265	19.9286	23.1622	18.7618	22.6147
1985	20.7483	16.9436	20.3429	22.6312	18.7498	22.4278
1986	20.7294	16.9265	20.3530	22.5904	18.8266	22.3586
1987	21.2076	17.0818	20.6653	23.9776	19.3790	23.2859
1988	21.9431	17.3971	21.1483	24.4038	19.7025	23.5507
1989	21.3069	17.2215	20.7920	23.5459	19.5700	23.3294
1990	21.4994	17.1843	20.8154	24.1807	19.6895	23.4925
1991	21.8167	17.4039	21.0742	23.5246	19.4685	23.2638
1992	18.0353	15.9672	18.3643	20.9543	18.5309	21.2384
1993	18.6202	16.2304	18.8946	28.2954	20.3039	26.2111

1. Moments are averages over 10^8 synthetic observations.
2. Percentiles from distribution of 10^4 sample averages, each based on 10^4 observations.

Table 3. Decomposition of $\ln[E(Y)]$ as given by $\ln[G_{\beta 1}(Y)]$

	Pulp and paper					Basic metals				
Year	$\ln[\psi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_{\beta 1})$	$\ln[E(Y)]$	Simulation	$\ln[\psi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_{\beta 1})$	$\ln[E(Y)]$	Simulation
1972	5.8221	-0.1336	0.2241	5.9126	5.9642	6.5750	-0.1186	0.2111	6.6675	6.7566
1973	5.8886	-0.1301	0.2300	5.9885	6.0421	6.6588	-0.1394	0.2302	6.7497	6.8375
1974	6.0399	-0.1524	0.3090	6.1965	6.2679	6.6042	-0.1211	0.1863	6.6694	6.7475
1975	5.9314	-0.1559	0.3047	6.0802	6.1488	6.7501	-0.1205	0.2440	6.8736	6.9647
1976	5.9413	-0.1622	0.3085	6.0876	6.1558	6.5689	-0.1446	0.2158	6.6401	6.7192
1977	5.8042	-0.1485	0.2434	5.8991	5.9498	6.6196	-0.1544	0.2387	6.7038	6.7825
1978	5.8231	-0.1493	0.2450	5.9188	5.9708	6.3791	-0.1339	0.1805	6.4258	6.4945
1979	5.9353	-0.1263	0.2336	6.0426	6.0949	6.7926	-0.1217	0.2084	6.8793	6.9541
1980	5.9900	-0.1171	0.2125	6.0854	6.1343	6.8043	-0.1425	0.1984	6.8602	6.9318
1981	6.0488	-0.1184	0.2246	6.1550	6.2062	6.7392	-0.1455	0.1898	6.7836	6.8530
1982	6.0927	-0.1067	0.2440	6.2301	6.2892	6.6320	-0.1415	0.1793	6.6698	6.7398
1983	6.1363	-0.0792	0.1826	6.2397	6.2855	6.7103	-0.1255	0.1757	6.7605	6.8360
1984	6.1470	-0.0747	0.1779	6.2502	6.2953	6.9263	-0.1300	0.1756	6.9718	7.0430
1985	6.2474	-0.0752	0.1854	6.3577	6.4049	6.8837	-0.1081	0.1482	6.9238	7.0030
1986	6.2395	-0.0844	0.1923	6.3475	6.3942	6.9838	-0.1014	0.1438	7.0261	7.0995
1987	6.2426	-0.0742	0.2013	6.3696	6.4191	7.1013	-0.1042	0.1664	7.1635	7.2459
1988	6.2859	-0.0641	0.2126	6.4344	6.4893	7.2176	-0.0732	0.1591	7.3036	7.3868
1989	6.2919	-0.0647	0.1961	6.4232	6.4741	7.1698	-0.0767	0.1480	7.2411	7.3270
1990	6.2698	-0.0790	0.2047	6.3955	6.4467	7.2243	-0.0766	0.1528	7.3004	7.3848
1991	6.3335	-0.0687	0.2040	6.4688	6.5224	7.2103	-0.1095	0.1645	7.2653	7.3421
1992	6.1708	-0.0398	0.1114	6.2424	6.2779	7.0697	-0.0712	0.0941	7.0927	7.1605
1993	6.2397	-0.0725	0.1334	6.3007	6.3373	7.1142	-0.3304	0.2801	7.0639	7.2189

Table 4. Decomposition of $\ln[E(Y^2)]$ as given by $\ln[G_{\beta 2}(Y)]$

	Pulp and paper					Basic metals				
Year	$\ln[\psi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_{\beta 2})$	$\ln[E(Y)]$	Simulation	$\ln[\psi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_{\beta 2})$	$\ln[E(Y)]$	Simulation
1972	15.7045	-1.0688	3.5857	18.2214	20.3222	20.1521	-0.9489	3.3775	22.5807	23.6420
1973	15.8728	-1.0408	3.6798	18.5118	20.7355	20.5013	-1.1149	3.6837	23.0702	24.3111
1974	16.7528	-1.2192	4.9443	20.4779	23.3204	19.8479	-0.9692	2.9816	21.8603	23.0945
1975	16.4704	-1.2468	4.8747	20.0983	22.9392	20.9268	-0.9639	3.9046	23.8675	25.0652
1976	16.5261	-1.2974	4.9357	20.1643	23.0748	20.0598	-1.1567	3.4520	22.3552	23.6211
1977	15.7070	-1.1880	3.8939	18.4128	20.7370	20.3634	-1.2352	3.8186	22.9468	24.0757
1978	15.7740	-1.1946	3.9203	18.4996	20.7938	19.2740	-1.0711	2.8884	21.0912	22.1762
1979	15.9206	-1.0106	3.7379	18.6479	20.9894	20.4934	-0.9736	3.3345	22.8542	23.9905
1980	15.8562	-0.9369	3.3999	18.3193	20.5820	20.2736	-1.1398	3.1746	22.3084	23.4015
1981	16.0864	-0.9472	3.5935	18.7327	21.1147	20.0626	-1.1638	3.0372	21.9360	22.9574
1982	16.3489	-0.8535	3.9044	19.3998	21.9550	19.7675	-1.1319	2.8683	21.5039	22.5748
1983	15.9355	-0.6338	2.9221	18.2239	20.4060	19.8954	-1.0042	2.8118	21.7029	23.0892
1984	15.9089	-0.5973	2.8457	18.1573	20.2310	20.2629	-1.0404	2.8094	22.0319	23.1622
1985	16.1904	-0.6013	2.9670	18.5561	20.7483	19.7910	-0.8651	2.3718	21.2977	22.6312
1986	16.1914	-0.6748	3.0775	18.5940	20.7294	19.9254	-0.8115	2.3001	21.4140	22.5904
1987	16.2786	-0.5938	3.2204	18.9051	21.2076	20.5655	-0.8332	2.6616	22.3939	23.9776
1988	16.4840	-0.5127	3.4019	19.3733	21.9431	20.6914	-0.5853	2.5458	22.6519	24.4038
1989	16.3544	-0.5177	3.1375	18.9742	21.3069	20.3009	-0.6138	2.3672	22.0542	23.5459
1990	16.3813	-0.6320	3.2756	19.0249	21.4994	20.5871	-0.6129	2.4445	22.4187	24.1807
1991	16.5299	-0.5499	3.2642	19.2442	21.8167	20.5945	-0.8760	2.6322	22.3508	23.5246
1992	15.2931	-0.3184	1.7822	16.7569	18.0353	19.0666	-0.5692	1.5062	20.0036	20.9543
1993	15.6447	-0.5797	2.1349	17.1999	18.6202	19.8738	-2.6436	4.4814	21.7116	28.2954

Table 5. Decomposition of ln[E(Y)] as given by $\ln[G_{x1}(Y)]$

	Pulp and paper					Basic metals				
Year	$\ln[\psi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_{x1})$	ln[E(Y)]	Simulation	$\ln[\psi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_{x1})$	ln[E(Y)]	Simulation
1972	5.8221	-0.1336	0.0081	5.6967	5.9642	6.5750	-0.1186	0.0059	6.4623	6.7566
1973	5.8886	-0.1301	0.0082	5.7667	6.0421	6.6588	-0.1394	0.0072	6.5267	6.8375
1974	6.0399	-0.1524	0.0079	5.8954	6.2679	6.6042	-0.1211	0.0062	6.4892	6.7475
1975	5.9314	-0.1559	0.0066	5.7822	6.1488	6.7501	-0.1205	0.0059	6.6355	6.9647
1976	5.9413	-0.1622	0.0067	5.7858	6.1558	6.5689	-0.1446	0.0070	6.4313	6.7192
1977	5.8042	-0.1485	0.0055	5.6612	5.9498	6.6196	-0.1544	0.0055	6.4706	6.7825
1978	5.8231	-0.1493	0.0062	5.6799	5.9708	6.3791	-0.1339	0.0047	6.2499	6.4946
1979	5.9353	-0.1263	0.0060	5.8149	6.0949	6.7926	-0.1217	0.0052	6.6760	6.9541
1980	5.9900	-0.1171	0.0067	5.8795	6.1343	6.8043	-0.1425	0.0075	6.6692	6.9318
1981	6.0488	-0.1184	0.0063	5.9368	6.2062	6.7392	-0.1455	0.0063	6.6001	6.8530
1982	6.0927	-0.1067	0.0059	5.9919	6.2892	6.6320	-0.1415	0.0071	6.4976	6.7398
1983	6.1363	-0.0792	0.0056	6.0626	6.2855	6.7103	-0.1255	0.0089	6.5937	6.8360
1984	6.1470	-0.0747	0.0053	6.0777	6.2953	6.9263	-0.1300	0.0094	6.8057	7.0430
1985	6.2474	-0.0752	0.0062	6.1784	6.4049	6.8837	-0.1081	0.0119	6.7874	7.0030
1986	6.2395	-0.0844	0.0054	6.1605	6.3942	6.9838	-0.1014	0.0104	6.8927	7.0996
1987	6.2426	-0.0742	0.0041	6.1724	6.4191	7.1013	-0.1042	0.0121	7.0093	7.2459
1988	6.2859	-0.0641	0.0044	6.2262	6.4893	7.2176	-0.0732	0.0108	7.1552	7.3868
1989	6.2919	-0.0647	0.0047	6.2319	6.4741	7.1698	-0.0767	0.0147	7.1078	7.3270
1990	6.2698	-0.0790	0.0054	6.1962	6.4467	7.2243	-0.0766	0.0132	7.1609	7.3848
1991	6.3335	-0.0687	0.0061	6.2709	6.5224	7.2103	-0.1095	0.0117	7.1125	7.3421
1992	6.1708	-0.0398	0.0045	6.1355	6.2779	7.0697	-0.0712	0.0127	7.0112	7.1605
1993	6.2397	-0.0725	0.0056	6.1729	6.3373	7.1142	-0.3304	0.0720	6.8558	7.2189

Table 6. Decomposition of $\ln[E(Y^2)]$ as given by $\ln[G_{x2}(Y)]$

	Pulp and paper					Basic metals				
Year	$\ln[\psi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_{x2})$	$\ln[E(Y)]$	Simulation	$\ln[\psi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_{x2})$	$\ln[E(Y)]$	Simulation
1972	15.7045	-1.0688	0.1301	14.7658	20.3222	20.1521	-0.9489	0.0951	19.2983	23.6420
1973	15.8728	-1.0408	0.1310	14.9630	20.7355	20.5013	-1.1149	0.1158	19.5022	24.3112
1974	16.7528	-1.2192	0.1261	15.6597	23.3204	19.8479	-0.9692	0.0986	18.9774	23.0945
1975	16.4704	-1.2468	0.1062	15.3298	22.9392	20.9268	-0.9639	0.0939	20.0568	25.0652
1976	16.5261	-1.2974	0.1074	15.3361	23.0748	20.0598	-1.1567	0.1118	19.0150	23.6211
1977	15.7070	-1.1880	0.0879	14.6069	20.7370	20.3634	-1.2352	0.0875	19.2157	24.0757
1978	15.7740	-1.1946	0.0990	14.6783	20.7938	19.2740	-1.0711	0.0745	18.2773	22.1762
1979	15.9206	-1.0106	0.0953	15.0053	20.9894	20.4934	-0.9736	0.0825	19.6023	23.9905
1980	15.8562	-0.9369	0.1065	15.0258	20.5820	20.2736	-1.1398	0.1194	19.2532	23.4015
1981	16.0864	-0.9472	0.1010	15.2402	21.1147	20.0626	-1.1638	0.1016	19.0004	22.9574
1982	16.3489	-0.8535	0.0942	15.5896	21.9550	19.7675	-1.1319	0.1144	18.7500	22.5748
1983	15.9355	-0.6338	0.0894	15.3911	20.4060	19.8954	-1.0042	0.1429	19.0341	23.0892
1984	15.9089	-0.5973	0.0853	15.3969	20.2310	20.2629	-1.0404	0.1510	19.3736	23.1622
1985	16.1904	-0.6013	0.0996	15.6887	20.7483	19.7910	-0.8651	0.1901	19.1161	22.6312
1986	16.1914	-0.6748	0.0858	15.6024	20.7294	19.9254	-0.8115	0.1656	19.2795	22.5904
1987	16.2786	-0.5938	0.0659	15.7506	21.2076	20.5655	-0.8332	0.1930	19.9253	23.9776
1988	16.4840	-0.5127	0.0706	16.0420	21.9431	20.6914	-0.5853	0.1723	20.2784	24.4038
1989	16.3544	-0.5177	0.0759	15.9126	21.3069	20.3009	-0.6138	0.2348	19.9218	23.5459
1990	16.3813	-0.6320	0.0871	15.8364	21.4994	20.5871	-0.6129	0.2115	20.1857	24.1807
1991	16.5299	-0.5499	0.0979	16.0780	21.8167	20.5945	-0.8760	0.1869	19.9054	23.5246
1992	15.2931	-0.3184	0.0722	15.0469	18.0353	19.0666	-0.5692	0.2028	18.7002	20.9543
1993	15.6447	-0.5797	0.0901	15.1550	18.6202	19.8738	-2.6436	1.1523	18.3825	23.6895

Table 7. Decomposition of $\ln[\psi_1(y)]$

Year	Pulp and paper						Basic metals					
	μ_y	$\frac{1}{2}\mu_x\Sigma_{\beta\beta}\mu_x^{/}$	$\frac{1}{2}\mu_\beta\Sigma_{xx}\mu_\beta^{/}$	$\frac{1}{2}tr(\Sigma_{\beta\beta}\Sigma_{xx})$	$\frac{1}{2}\sigma^2$	$\ln[\psi_1(y)]$	μ_y	$\frac{1}{2}\mu_x\Sigma_{\beta\beta}\mu_x^{/}$	$\frac{1}{2}\mu_\beta\Sigma_{xx}\mu_\beta^{/}$	$\frac{1}{2}tr(\Sigma_{\beta\beta}\Sigma_{xx})$	$\frac{1}{2}\sigma^2$	$\ln[\psi_1(y)]$
1972	3.7920	1.6622	0.2131	0.1344	0.0204	5.8221	3.0739	2.9927	0.2677	0.1913	0.0493	6.5750
1973	3.8408	1.6760	0.2161	0.1353	0.0204	5.8886	3.0670	3.0811	0.2773	0.1842	0.0493	6.6588
1974	3.7034	1.9610	0.2159	0.1392	0.0204	6.0399	3.2845	2.8307	0.2758	0.1639	0.0493	6.6042
1975	3.6275	1.9477	0.1973	0.1384	0.0204	5.9314	3.0368	3.2270	0.2596	0.1774	0.0493	6.7501
1976	3.6195	1.9655	0.1959	0.1400	0.0204	5.9413	3.1078	2.9886	0.2574	0.1658	0.0493	6.5689
1977	3.7550	1.7172	0.1869	0.1248	0.0204	5.8042	3.0574	3.1199	0.2403	0.1527	0.0493	6.6196
1978	3.7592	1.7227	0.1931	0.1277	0.0204	5.8231	3.1213	2.8368	0.2380	0.1336	0.0493	6.3791
1979	3.9103	1.6803	0.1985	0.1257	0.0204	5.9353	3.3385	3.0251	0.2414	0.1383	0.0493	6.7926
1980	4.0518	1.5885	0.1983	0.1309	0.0204	5.9900	3.4717	2.8882	0.2526	0.1425	0.0493	6.8043
1981	4.0545	1.6453	0.1993	0.1293	0.0204	6.0488	3.4472	2.8606	0.2431	0.1391	0.0493	6.7392
1982	4.0110	1.7312	0.1949	0.1353	0.0204	6.0927	3.3802	2.8161	0.2473	0.1391	0.0493	6.6320
1983	4.3047	1.4942	0.1954	0.1216	0.0204	6.1363	3.4730	2.7874	0.2487	0.1520	0.0493	6.7103
1984	4.3396	1.4732	0.1999	0.1139	0.0204	6.1470	3.7212	2.7609	0.2556	0.1394	0.0493	6.9263
1985	4.3996	1.5023	0.2098	0.1153	0.0204	6.2474	3.8718	2.5396	0.2647	0.1582	0.0493	6.8837
1986	4.3833	1.5183	0.2047	0.1128	0.0204	6.2395	4.0049	2.5266	0.2612	0.1418	0.0493	6.9838
1987	4.3458	1.5714	0.1954	0.1096	0.0204	6.2426	3.9199	2.7059	0.2730	0.1532	0.0493	7.1013
1988	4.3297	1.6227	0.1939	0.1192	0.0204	6.2859	4.0896	2.6569	0.2745	0.1474	0.0493	7.2176
1989	4.4065	1.5481	0.2001	0.1167	0.0204	6.2919	4.1892	2.4672	0.2967	0.1674	0.0493	7.1698
1990	4.3489	1.5767	0.2032	0.1205	0.0204	6.2698	4.1550	2.5806	0.2910	0.1484	0.0493	7.2243
1991	4.4020	1.5806	0.2023	0.1282	0.0204	6.3335	4.1233	2.5909	0.2928	0.1540	0.0493	7.2103
1992	4.6950	1.1542	0.1989	0.1023	0.0204	6.1708	4.6061	1.9923	0.2986	0.1234	0.0493	7.0697
1993	4.6571	1.2376	0.2178	0.1068	0.0204	6.2397	4.2915	2.0502	0.3946	0.3285	0.0493	7.1142

Table 8. Decomposition of $\ln[\psi_2(y)]$

Year	Pulp and paper						Basic metals					
	$2\mu_y$	$2\mu_x\Sigma_{\beta\beta}\mu_x^{/}$	$2\mu_\beta\Sigma_{xx}\mu_\beta^{/}$	$2tr(\Sigma_{\beta\beta}\Sigma_{xx})$	$2\sigma^2$	$\ln[\psi_2(y)]$	$2\mu_y$	$2\mu_x\Sigma_{\beta\beta}\mu_x^{/}$	$2\mu_\beta\Sigma_{xx}\mu_\beta^{/}$	$2tr(\Sigma_{\beta\beta}\Sigma_{xx})$	$2\sigma^2$	$\ln[\psi_2(y)]$
1972	7.5840	6.6488	0.8526	0.5375	0.0816	15.7045	6.1478	11.9709	1.0709	0.7652	0.1972	20.1521
1973	7.6815	6.7040	0.8645	0.5411	0.0816	15.8728	6.1339	12.3246	1.1090	0.7366	0.1972	20.5013
1974	7.4067	7.8440	0.8636	0.5570	0.0816	16.7528	6.5690	11.3230	1.1031	0.6557	0.1972	19.8479
1975	7.2550	7.7910	0.7893	0.5535	0.0816	16.4704	6.0735	12.9080	1.0383	0.7098	0.1972	20.9268
1976	7.2390	7.8619	0.7836	0.5600	0.0816	16.5261	6.2157	11.9544	1.0294	0.6631	0.1972	20.0598
1977	7.5100	6.8690	0.7474	0.4990	0.0816	15.7070	6.1148	12.4794	0.9611	0.6108	0.1972	20.3634
1978	7.5184	6.8908	0.7724	0.5108	0.0816	15.7740	6.2426	11.3473	0.9522	0.5346	0.1972	19.2740
1979	7.8206	6.7213	0.7942	0.5029	0.0816	15.9206	6.6770	12.1006	0.9655	0.5531	0.1972	20.4934
1980	8.1037	6.3540	0.7934	0.5235	0.0816	15.8562	6.9435	11.5526	1.0102	0.5700	0.1972	20.2736
1981	8.1089	6.5813	0.7973	0.5173	0.0816	16.0864	6.8943	11.4424	0.9723	0.5564	0.1972	20.0626
1982	8.0220	6.9247	0.7796	0.5411	0.0816	16.3489	6.7604	11.2646	0.9891	0.5562	0.1972	19.7675
1983	8.6095	5.9766	0.7815	0.4863	0.0816	15.9355	6.9459	11.1495	0.9949	0.6079	0.1972	19.8954
1984	8.6792	5.8929	0.7995	0.4558	0.0816	15.9089	7.4423	11.0436	1.0222	0.5576	0.1972	20.2629
1985	8.7991	6.0093	0.8390	0.4613	0.0816	16.1904	7.7436	10.1585	1.0588	0.6329	0.1972	19.7910
1986	8.7666	6.0732	0.8187	0.4513	0.0816	16.1914	8.0098	10.1064	1.0448	0.5672	0.1972	19.9254
1987	8.6917	6.2854	0.7816	0.4383	0.0816	16.2786	7.8399	10.8236	1.0921	0.6126	0.1972	20.5655
1988	8.6595	6.4907	0.7755	0.4768	0.0816	16.4840	8.1791	10.6276	1.0980	0.5895	0.1972	20.6914
1989	8.8130	6.1926	0.8003	0.4669	0.0816	16.3544	8.3785	9.8688	1.1868	0.6695	0.1972	20.3009
1990	8.6978	6.3070	0.8130	0.4820	0.0816	16.3813	8.3099	10.3224	1.1641	0.5935	0.1972	20.5871
1991	8.8041	6.3223	0.8094	0.5126	0.0816	16.5299	8.2466	10.3635	1.1712	0.6160	0.1972	20.5945
1992	9.3900	4.6168	0.7957	0.4090	0.0816	15.2931	9.2122	7.9692	1.1945	0.4935	0.1972	19.0666
1993	9.3142	4.9504	0.8711	0.4274	0.0816	15.6447	8.5831	8.2007	1.5786	1.3142	0.1972	19.8738

Table 9. Dispersion preserving macro input and scale elasticities based on $\ln[G_{\beta 1}(Y)]$

	Capital elasticity	Labour elasticity	Energy elasticity	Materials elasticity	Scale elasticity
Pulp and paper					
1972	0.532	0.883	-0.028	0.390	1.778
1973	0.529	0.876	-0.022	0.394	1.777
1974	0.523	0.884	-0.022	0.411	1.796
1975	0.535	0.881	-0.023	0.398	1.791
1976	0.542	0.871	-0.024	0.399	1.789
1977	0.564	0.882	-0.028	0.364	1.782
1978	0.557	0.871	-0.025	0.374	1.776
1979	0.545	0.866	-0.022	0.386	1.775
1980	0.552	0.864	-0.026	0.383	1.774
1981	0.555	0.850	-0.025	0.392	1.772
1982	0.544	0.842	-0.023	0.410	1.772
1983	0.558	0.837	-0.028	0.397	1.765
1984	0.552	0.835	-0.026	0.401	1.763
1985	0.550	0.830	-0.024	0.408	1.764
1986	0.557	0.838	-0.024	0.398	1.769
1987	0.555	0.834	-0.024	0.405	1.769
1988	0.552	0.817	-0.020	0.415	1.764
1989	0.553	0.817	-0.021	0.412	1.761
1990	0.555	0.817	-0.021	0.410	1.761
1991	0.557	0.810	-0.018	0.412	1.760
1992	0.566	0.804	-0.025	0.392	1.738
1993	0.570	0.807	-0.025	0.393	1.744
Basic metals					
1972	0.068	0.914	0.007	0.503	1.491
1973	0.088	0.912	-0.016	0.515	1.498
1974	0.116	0.878	-0.030	0.524	1.488
1975	0.103	0.879	-0.027	0.538	1.494
1976	0.108	0.880	-0.010	0.506	1.485
1977	0.109	0.881	0.000	0.495	1.485
1978	0.156	0.819	-0.012	0.500	1.463
1979	0.132	0.828	-0.004	0.515	1.471
1980	0.132	0.844	-0.003	0.500	1.473
1981	0.166	0.829	-0.016	0.491	1.471
1982	0.200	0.794	-0.034	0.503	1.462
1983	0.185	0.778	-0.023	0.515	1.455
1984	0.175	0.791	-0.016	0.510	1.459
1985	0.213	0.772	-0.055	0.529	1.460
1986	0.203	0.767	-0.038	0.523	1.455
1987	0.219	0.756	-0.060	0.545	1.460
1988	0.209	0.743	-0.059	0.563	1.457
1989	0.199	0.739	-0.061	0.576	1.452
1990	0.204	0.728	-0.052	0.568	1.449
1991	0.183	0.741	-0.028	0.551	1.446
1992	0.213	0.698	-0.025	0.537	1.424
1993	0.122	0.738	0.024	0.537	1.421

Table 10. Dispersion preserving macro input and scale elasticities based on $\ln[G_{x1}(Y)]$

	Capital elasticity	Labour elasticity	Energy elasticity	Materials elasticity	Scale elasticity
Pulp and paper					
1972	0.559	0.926	-0.031	0.382	1.836
1973	0.556	0.917	-0.025	0.385	1.832
1974	0.547	0.927	-0.023	0.406	1.857
1975	0.558	0.923	-0.023	0.392	1.850
1976	0.566	0.916	-0.026	0.393	1.850
1977	0.588	0.929	-0.031	0.357	1.844
1978	0.580	0.920	-0.028	0.367	1.839
1979	0.570	0.917	-0.026	0.377	1.838
1980	0.581	0.925	-0.031	0.367	1.841
1981	0.581	0.906	-0.029	0.378	1.836
1982	0.566	0.892	-0.027	0.401	1.832
1983	0.587	0.892	-0.033	0.381	1.827
1984	0.580	0.886	-0.031	0.387	1.821
1985	0.579	0.881	-0.029	0.394	1.825
1986	0.583	0.890	-0.029	0.386	1.831
1987	0.576	0.878	-0.027	0.400	1.826
1988	0.572	0.863	-0.023	0.408	1.820
1989	0.574	0.861	-0.023	0.404	1.816
1990	0.577	0.864	-0.023	0.400	1.818
1991	0.582	0.860	-0.021	0.398	1.819
1992	0.588	0.850	-0.029	0.378	1.788
1993	0.596	0.857	-0.030	0.377	1.800
Basic metals					
1972	0.070	0.942	-0.009	0.508	1.511
1973	0.099	0.938	-0.046	0.530	1.522
1974	0.132	0.899	-0.059	0.537	1.509
1975	0.117	0.901	-0.058	0.556	1.515
1976	0.118	0.903	-0.033	0.516	1.504
1977	0.110	0.913	-0.016	0.500	1.506
1978	0.161	0.842	-0.026	0.502	1.479
1979	0.134	0.855	-0.016	0.515	1.488
1980	0.129	0.877	-0.011	0.497	1.492
1981	0.169	0.856	-0.029	0.492	1.489
1982	0.208	0.815	-0.050	0.506	1.479
1983	0.193	0.797	-0.039	0.519	1.470
1984	0.183	0.810	-0.030	0.512	1.474
1985	0.229	0.786	-0.081	0.543	1.477
1986	0.211	0.778	-0.057	0.536	1.469
1987	0.236	0.766	-0.086	0.559	1.476
1988	0.226	0.752	-0.086	0.580	1.471
1989	0.205	0.754	-0.086	0.594	1.468
1990	0.216	0.740	-0.081	0.589	1.465
1991	0.194	0.754	-0.055	0.568	1.461
1992	0.224	0.707	-0.051	0.555	1.435
1993	0.228	0.689	-0.069	0.579	1.428

Table 11. Mean preserving macro input and scale elasticities based on $\ln[G_{\beta 1}(Y)]$

	Capital elasticity	Labour elasticity	Energy elasticity	Materials elasticity	Scale elasticity
Pulp and paper					
1972	0.318	0.425	0.011	0.227	0.982
1973	0.317	0.423	0.012	0.231	0.983
1974	0.314	0.426	0.012	0.250	1.001
1975	0.320	0.425	0.012	0.235	0.992
1976	0.323	0.421	0.012	0.237	0.993
1977	0.334	0.425	0.011	0.197	0.967
1978	0.331	0.421	0.012	0.208	0.972
1979	0.325	0.420	0.012	0.221	0.978
1980	0.328	0.419	0.012	0.219	0.977
1981	0.330	0.414	0.012	0.228	0.984
1982	0.324	0.411	0.012	0.249	0.996
1983	0.332	0.410	0.011	0.234	0.986
1984	0.329	0.409	0.012	0.238	0.987
1985	0.327	0.407	0.012	0.247	0.993
1986	0.331	0.410	0.012	0.235	0.987
1987	0.330	0.408	0.012	0.243	0.993
1988	0.328	0.403	0.013	0.255	0.998
1989	0.329	0.403	0.012	0.251	0.995
1990	0.330	0.403	0.012	0.249	0.994
1991	0.331	0.400	0.013	0.251	0.995
1992	0.335	0.398	0.012	0.229	0.974
1993	0.337	0.399	0.012	0.229	0.977
Basic metals					
1972	0.162	0.609	0.076	0.373	1.219
1973	0.167	0.607	0.066	0.385	1.225
1974	0.174	0.589	0.061	0.393	1.217
1975	0.171	0.590	0.062	0.407	1.229
1976	0.172	0.590	0.069	0.376	1.207
1977	0.172	0.590	0.073	0.365	1.200
1978	0.184	0.557	0.068	0.370	1.179
1979	0.178	0.562	0.071	0.385	1.195
1980	0.178	0.570	0.072	0.370	1.190
1981	0.186	0.562	0.067	0.362	1.176
1982	0.195	0.542	0.059	0.373	1.169
1983	0.191	0.534	0.063	0.385	1.173
1984	0.189	0.541	0.066	0.379	1.175
1985	0.198	0.531	0.050	0.399	1.177
1986	0.195	0.528	0.057	0.393	1.173
1987	0.199	0.522	0.047	0.415	1.183
1988	0.197	0.515	0.048	0.432	1.192
1989	0.194	0.512	0.047	0.444	1.198
1990	0.196	0.507	0.051	0.437	1.191
1991	0.190	0.514	0.061	0.420	1.185
1992	0.198	0.490	0.063	0.406	1.157
1993	0.175	0.512	0.083	0.407	1.178

Table 12. Mean preserving macro input and scale elasticities based on $\ln[G_{x1}(Y)]$

	Capital elasticity	Labour elasticity	Energy elasticity	Materials elasticity	Scale elasticity
Pulp and paper					
1972	0.354	0.827	0.025	0.229	1.436
1973	0.352	0.826	0.025	0.233	1.435
1974	0.355	0.818	0.025	0.228	1.426
1975	0.374	0.819	0.025	0.216	1.434
1976	0.379	0.803	0.025	0.215	1.422
1977	0.385	0.804	0.025	0.214	1.428
1978	0.386	0.779	0.025	0.218	1.407
1979	0.376	0.781	0.025	0.226	1.408
1980	0.380	0.767	0.025	0.231	1.403
1981	0.383	0.759	0.025	0.233	1.401
1982	0.392	0.755	0.025	0.230	1.402
1983	0.388	0.755	0.025	0.237	1.405
1984	0.382	0.757	0.025	0.241	1.405
1985	0.377	0.745	0.025	0.247	1.394
1986	0.382	0.743	0.025	0.243	1.393
1987	0.392	0.746	0.025	0.237	1.399
1988	0.396	0.742	0.025	0.236	1.400
1989	0.393	0.735	0.025	0.242	1.395
1990	0.393	0.730	0.025	0.242	1.390
1991	0.396	0.723	0.025	0.241	1.385
1992	0.398	0.730	0.025	0.245	1.398
1993	0.384	0.718	0.025	0.255	1.382
Basic metals					
1972	0.171	0.851	0.119	0.357	1.499
1973	0.174	0.823	0.120	0.372	1.489
1974	0.173	0.828	0.119	0.372	1.493
1975	0.182	0.807	0.120	0.365	1.475
1976	0.187	0.783	0.121	0.368	1.460
1977	0.187	0.767	0.119	0.345	1.419
1978	0.187	0.760	0.119	0.340	1.406
1979	0.187	0.755	0.119	0.350	1.411
1980	0.190	0.739	0.120	0.366	1.416
1981	0.191	0.742	0.120	0.355	1.408
1982	0.198	0.717	0.120	0.358	1.394
1983	0.207	0.693	0.121	0.357	1.377
1984	0.204	0.692	0.121	0.370	1.387
1985	0.207	0.682	0.122	0.381	1.392
1986	0.207	0.682	0.122	0.380	1.390
1987	0.211	0.664	0.122	0.385	1.382
1988	0.209	0.665	0.122	0.390	1.386
1989	0.222	0.631	0.124	0.403	1.380
1990	0.217	0.637	0.123	0.399	1.377
1991	0.223	0.627	0.124	0.395	1.370
1992	0.222	0.633	0.125	0.412	1.391
1993	0.289	0.549	0.140	0.430	1.408

APPENDIX A: Conditions for the existence of origo moments

In this Appendix, we prove condition (20), which ensures the existence of origo moments of output. We also show that if the moment of order r exists, then all lower-order moments also exist. Rearranging (17), we find

$$\lambda_{\beta r} = -\tfrac{1}{2}[\beta'(\Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx})\beta - 2\mu_\beta'\Sigma_{\beta\beta}^{-1}\beta - 2r\mu_x\beta + \mu_\beta'\Sigma_{\beta\beta}^{-1}\mu_\beta], \tag{A.1}$$

which can be simplified to

$$\lambda_{\beta r} = -\tfrac{1}{2}[\beta'(\Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx})\beta + a\beta + b],$$

where $a = -2(\mu_\beta'\Sigma_{\beta\beta}^{-1} + r\mu_x)$ and $b = \mu_\beta'\Sigma_{\beta\beta}^{-1}\mu_\beta$. Diagonalizing $\Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx}$ we obtain

$$\lambda_{\beta r} = -\tfrac{1}{2}[\beta' UDU'\beta + a\beta + b],$$

where U is an orthogonal matrix, and D is a diagonal matrix with the eigenvalues of

$$M(r) = \Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx}, \tag{A.2}$$

denoted λ_i, on the main diagonal. Using the linear transformation $\widetilde{\beta} = U'\beta$, $\widetilde{a} = a(U')^{-1} = aU$, we can write the last expression as

$$\lambda_{\beta r} = -\tfrac{1}{2}[\widetilde{\beta}' D\widetilde{\beta} + \widetilde{a}\widetilde{\beta} + b], \tag{A.3}$$

or, when letting $\widetilde{\beta} = (\widetilde{\beta}_1, \ldots, \widetilde{\beta}_{n+1})$ and $\widetilde{a} = (\widetilde{a}_1, \ldots, \widetilde{a}_{n+1})$, as

$$\lambda_{\beta r} = -\tfrac{1}{2}\left[\sum_i \lambda_i\widetilde{\beta}_i^2 + \sum_i \widetilde{a}_i\widetilde{\beta}_i + b\right] = -\tfrac{1}{2}\left[\sum_i \lambda_i\left(\widetilde{\beta}_i + \frac{\widetilde{a}_i}{2\lambda_i}\right)^2 + \widetilde{b}\right], \tag{A.4}$$

where $\widetilde{b} = b - \sum_i \widetilde{a}_i{}^2/(4\lambda_i^2)$. The integral in (16) can now be expressed by

$$\begin{aligned}\int_{R^{n+1}} e^{\lambda_{\beta r}} d\beta &= \int_{R^{n+1}} \exp\left(-\tfrac{1}{2}\left[\sum_i \lambda_i\left(\widetilde{\beta}_i + \frac{\widetilde{a}_i}{2\lambda_i}\right)^2 + \widetilde{b}\right]\right) d\widetilde{\beta} \\ &= k\int_{R^{n+1}} \exp\left(-\sum_i \frac{\lambda_i}{2}\widehat{\beta}_i^2\right) d\widehat{\beta},\end{aligned}$$

where $\widehat{\beta}_i = \widetilde{\beta}_i + \widetilde{a}_i/(2\lambda_i)$ and $k = e^{\widetilde{b}/2}$. It is separable and can be written as

$$\int_{R^{n+1}} e^{\lambda_{\beta r}} d\beta = k\prod_{i=1}^{n+1}\int_R \exp\left(-\sum_i \frac{\lambda_i}{2}\widehat{\beta}_i^2\right) d\widehat{\beta}_i. \tag{A.5}$$

A necessary and sufficient condition for the existence of this multiple integral, is that *all eigenvalues of the matrix* $M(r) = \Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx}$ *are strictly positive*. A corresponding existence condition may be derived from (18) and (19), and says that *all eigenvalues of* $\Sigma_{zz}^{-1} - r^2\Sigma_{\gamma\gamma}$ *are strictly positive*. The latter is equivalent to the condition that all eigenvalues of $\Sigma_{\gamma\gamma}^{-1} - r^2\Sigma_{zz}$ are positive, since $A - B$ is a positive definite matrix if and and only if $B^{-1} - A^{-1}$ is positive definite [cf. Magnus and Neudecker (1988, Chapter 1, Theorem 24)].

If the moment of order r exists, then all lower-order moments also exist. To see this we observe that

$$\text{(A.6)} \qquad M(r-1) = M(r) + (2r-1)\Sigma_{xx}, \quad r = 2, 3, \dots\ .$$

If $M(r)$ and Σ_{xx} are positive definite, then $M(r-1)$ is also positive definite, since $2r > 1$ and the sum of two positive definite matrices is positive definite.

APPENDIX B: The exact aggregate input elasticities – proofs

The purpose of this Appendix is to prove eqs. (37) and (38). Differentiating the various terms in (36) with respect to μ_z', we get

$$\text{(B.1)} \qquad \frac{\partial \mu_y}{\partial \mu_z'} = \frac{\partial(\mu_x\mu_\beta)}{\partial \mu_z'} = \frac{\partial(\mu_z\mu_\gamma)}{\partial \mu_z'} = \mu_\gamma,$$

$$\text{(B.2)} \qquad \frac{\partial \sigma_{yy}}{\partial \mu_z'} = \frac{\partial(\mu_x\Sigma_{\beta\beta}\mu_x')}{\partial \mu_z'} = \frac{\partial(\mu_z\Sigma_{\gamma\gamma}\mu_z')}{\partial \mu_z'} = 2\Sigma_{\gamma\gamma}\mu_z',$$

$$\text{(B.3)} \qquad \frac{\partial(\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial \mu_z'} = \frac{\partial(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \mu_z'} = \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma,$$

$$\text{(B.4)} \qquad \frac{\partial(\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu_x')}{\partial \mu_z'} = \frac{\partial(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\Sigma_{\gamma\gamma}\mu_z')}{\partial \mu_z'} = 2\Sigma_{\gamma\gamma}\Sigma_{zz}\Sigma_{\gamma\gamma}\mu_z'.$$

Differentiation with respect to Σ_{zz} [using Lütkepohl (1996, Section 10.3.2, eqs. (2), (5) and (21))] yields

$$\text{(B.5)} \qquad \frac{\partial \sigma_{yy}}{\partial \Sigma_{zz}} = \frac{\partial(\mu_\beta'\Sigma_{xx}\mu_\beta)}{\partial \Sigma_{zz}} + \frac{\partial \mathrm{tr}(\Sigma_{\beta\beta}\Sigma_{xx})}{\partial \Sigma_{zz}} = \frac{\partial(\mu_\gamma'\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} + \frac{\partial \mathrm{tr}(\Sigma_{\gamma\gamma}\Sigma_{zz})}{\partial \Sigma_{zz}} = \mu_\gamma\mu_\gamma' + \Sigma_{\gamma\gamma},$$

$$\text{(B.6)} \qquad \frac{\partial(\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial \Sigma_{zz}} = \frac{\partial(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} = \frac{\partial \mathrm{tr}(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} = \mu_\gamma\mu_z\Sigma_{\gamma\gamma},$$

$$\text{(B.7)}\quad \frac{\partial(\mu'_\beta\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial\Sigma_{zz}} = \frac{\partial(\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial\Sigma_{zz}} = \frac{\partial \mathrm{tr}(\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial\Sigma_{zz}} = \mu_\gamma\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu'_\gamma,$$

$$\text{(B.8)}\quad \frac{\partial(\mu_x\Sigma_{\beta\beta}\Sigma_{xx}\Sigma_{\beta\beta}\mu'_x)}{\partial\Sigma_{zz}} = \frac{\partial(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\Sigma_{\gamma\gamma}\mu'_z)}{\partial\Sigma_{zz}} = \frac{\partial \mathrm{tr}(\mu_z\Sigma_{\gamma\gamma}\Sigma_{zz}\Sigma_{\gamma\gamma}\mu'_z)}{\partial\Sigma_{zz}} = \Sigma_{\gamma\gamma}\mu'_z\mu_z\Sigma_{\gamma\gamma}.$$

It follows from (36) and (B.1) – (B.8), that

$$\text{(B.9)}\quad \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \mu'_z} &= \mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma = \mu_\gamma + \Sigma_{\gamma\gamma}(\mu'_z + \Sigma_{zz}\mu_\gamma),\\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \mu'_z} &= \mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma + \Sigma_{\gamma\gamma}\Sigma_{zz}\Sigma_{\gamma\gamma}\mu'_z\\ &= (I + \Sigma_{\gamma\gamma}\Sigma_{zz})(\mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z),\end{aligned}$$

$$\text{(B.10)}\quad \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \Sigma_{zz}} &= \tfrac{1}{2}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma}) + \mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \tfrac{1}{2}(\mu_\gamma\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu'_\gamma),\\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \Sigma_{zz}} &= \tfrac{1}{2}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma}) + \mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \tfrac{1}{2}\Sigma_{\gamma\gamma}\mu'_z\mu_z\Sigma_{\gamma\gamma}.\end{aligned}$$

Since, from (30), $\Delta \ln[\mathsf{E}(Z)]' = \Delta(\mu'_z + \frac{1}{2}\sigma_{zz})$, we have

$$\text{(B.11)}\quad \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma \quad \text{when } \Sigma_{zz} \text{ is constant},\\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= (I + \Sigma_{\gamma\gamma}\Sigma_{zz})(\mu_\gamma + \Sigma_{\gamma\gamma}\mu'_z) \quad \text{when } \Sigma_{zz} \text{ is constant},\end{aligned}$$

$$\text{(B.12)}\quad \begin{aligned} \frac{\partial \ln[G_{\beta 1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mathrm{diagv}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma} + 2\mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \mu_\gamma\mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu'_\gamma)\\ &\qquad \text{when } \mu_z \text{ and the off-diagonal elements of } \Sigma_{zz} \text{ are constant},\\ \frac{\partial \ln[G_{x1}(Y)]}{\partial \ln[\mathsf{E}(Z)]'} &= \mathrm{diagv}(\mu_\gamma\mu'_\gamma + \Sigma_{\gamma\gamma} + 2\mu_\gamma\mu_z\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\mu'_z\mu_z\Sigma_{\gamma\gamma})\\ &\qquad \text{when } \mu_z \text{ and the off-diagonal elements of } \Sigma_{zz} \text{ are constant}.\end{aligned}$$

This completes the proof.

APPENDIX C: Data

The data are from the years 1972 – 1993 and represent two Norwegian manufacturing industries, *Pulp and paper* and *Basic metals*. Table C.1, classifying the observations by the number of years, and Table C.2, sorting the plants by the calendar year in which they are observed, shows the unbalanced structure of the data set. There is a negative trend in the number of plants for both industries.

The primary data source is the Manufacturing Statistics database of Statistics Norway, classified under the Standard Industrial Classification (SIC)-codes 341 Manufacture of paper and paper products (Pulp and paper, for short) and 37 Manufacture of basic metals (Basic metals, for short). Both plants with contiguous and non-contiguous time series are included.

In the description below, MS indicates plant data from the Manufacturing Statistics, NNA indicates that the data are from the Norwegian National Accounts and are identical for plants classified in the same National Account industry. We use price indices from NNA to deflate total material costs, gross investments and fire insurance values. The two latter variables are used to calculate data on capital stocks, cf. below.

Y: Output, 100 tonnes (MS)

$K = KB + KM$: Total capital stock (buildings/structures plus
machinery/transport equipment), 100 000 1991-NOK (MS,NNA)

L: Labour input, 100 man-hours (MS)

E: Energy input, 100 000 kWh, electricity plus fuels (excl. motor gasoline) (MS)

$M = CM/QM$: Input of materials (incl. motor gasoline), 100 000 1991-NOK (MS,NNA)

CM: Total material cost (incl. motor gasoline) (MS)

QM: Price of materials (incl. motor gasoline), 1991=1 (NNA)

Output: The plants in the Manufacturing Statistics are in general multi-output plants and report output of a number of products measured in both NOK and primarily tonnes or kg. For each plant, an aggregate output measure in tonnes is calculated. Hence, rather than representing output in the two industries by deflated sales, which may be affected by measurement errors [see Klette and Griliches (1996)], our output measures are actual output in physical units, which are in several respects preferable.

Capital stock: The calculations of capital stock data are based on the perpetual inventory method assuming constant depreciation rates. We combine plant data on gross investment with fire insurance values for each of the two categories Buildings and structures and Machinery and transport equipment from the MS. The data on investment and fire insurance are deflated using industry specific price indices of investment goods from the NNA (1991=1). The depreciation rate for Buildings and structures is 0.020, for Machinery and transport equipment, it is set to 0.040 in both industries. For further documentation of the data and the calculations, see Biørn, Lindquist and Skjerpen (2000, Section 4, and 2003).

Other inputs: From the MS get the number of man-hours used, total electricity consumption in kWh, the consumption of a number of fuels in various denominations, and total material costs in NOK for each plant. The different fuels are transformed to the common denominator kWh by using estimated average energy content of each fuel [Statistics Norway (1995, p. 124)]. This enables us to calculate aggregate energy use in kWh for each plant. For most plants, this energy aggregate is dominated by electricity. Total material costs is deflated by the price index (1991=1) of material inputs from the NNA. This price is identical for all plants classified in the same National Account industry.

TABLE C.1. NUMBER OF PLANTS CLASSIFIED BY NUMBER OF REPLICATIONS

p = no. of observations per plant, N_p = no. of plants observed p times

Industry	*Pulp & paper*		*Basic metals*	
p	N_p	$N_p p$	N_p	$N_p p$
22	60	1320	44	968
21	9	189	2	42
20	5	100	4	80
19	3	57	5	95
18	1	18	2	36
17	4	68	5	85
16	6	96	5	80
15	4	60	4	60
14	3	42	5	70
13	4	52	3	39
12	7	84	10	120
11	10	110	7	77
10	12	120	6	60
09	10	90	5	45
08	7	56	2	16
07	15	105	13	91
06	11	66	4	24
05	14	70	5	25
04	9	36	6	24
03	18	54	3	9
02	5	10	6	12
01	20	20	20	20
Sum	237	2823	166	2078

Table C.2. Number of plants by calendar year

Year	*Pulp & paper*	*Basic metals*
1972	171	102
1973	171	105
1974	179	105
1975	175	110
1976	172	109
1977	158	111
1978	155	109
1979	146	102
1980	144	100
1981	137	100
1982	129	99
1983	111	95
1984	108	87
1985	106	89
1986	104	84
1987	102	87
1988	100	85
1989	97	83
1990	99	81
1991	95	81
1992	83	71
1993	81	83
Sum	2823	2078

We have removed observations with missing values of output or inputs. This reduced the number of observations by 6 – 8 per cent in the three industries.

APPENDIX D: Testing normality of log-output and log-inputs

In this Appendix, we present the results of formal univariate tests of whether, for each year in the sample period, log-output and log-inputs are normally distributed. The test statistic takes into account both skewness and excess kurtosis. The skewness and excess kurtosis test statistics are given by, respectively,

$$\text{(D.1)} \qquad \begin{aligned} T_S &= \sqrt{\frac{N}{6}}\frac{N^2}{(N-1)(N-2)}\frac{M_3}{M_2^{3/2}}, \\ T_K &= \sqrt{\frac{N}{24}}\frac{N^2}{(N-1)(N-2)(N-3)}\frac{(N+1)M_4-3(N-1)M_2^2}{M_2^2}, \end{aligned}$$

where N is the sample size and M_2, M_3 and M_4 are the centered second, third and fourth order sample moments. Both T_S and T_K are standard normally distributed under normality. Table 1 contains summary information on the results. In Tables D.2 and D.3 we test for skewness and excess kurtosis and report the two-tailed significance probabilities. Furthermore, it can be shown that T_S and T_K are asymptotically independent, which implies that

$$T_N = T_S^2 + T_K^2 \tag{D.2}$$

is χ^2-distributed with 2 degrees of freedom asymptotically [cf. Davidson and MacKinnon (1993, chapter 16.7) and Hall and Cummins (1999)]. The significance probabilities for the normality tests based on T_N are reported in Table D.1. If normality is rejected, Tables D.2 and D.3 show whether this is due to skewness and/or excess kurtosis.

For Pulp and paper, we find some evidence of non-normality, especially in the first years in the sample. Non-normality is most pronounced for energy and materials, and normality is rejected at the 1 per cent significance level for both inputs in the first five years. From Tables D.2 and D.3 skewness seems to be the reason for non-normality for energy, whereas non-normality for materials can be associated with excess kurtosis. For output and capital and labour inputs normality is never rejected at the 1 per cent significance level. For Basic metals, normality is not rejected for any of the inputs or output in any year using the 1 per cent significance level. Besides, at the 5 per cent significance level, normality is only rejected in two cases, for output in 1972 and for energy in the last year, 1993. From Table D.2 we see that the significance probability for the skewness tests are generally very high. However, Table D.3 reveals that there are some signs of excess kurtosis in this industry, especially for output and labour at the start of the sample period.

Table D.1. Testing for normality of log-output and log-input variables[1]

Year	log(X)	log(K)	log(L)	log(E)	log(M)
Pulp and paper					
1972	0.027	0.097	0.158	0.005	0.051
1973	0.017	0.055	0.176	0.003	0.063
1974	0.013	0.043	0.068	0.001	0.006
1975	0.016	0.049	0.124	0.003	0.001
1976	0.014	0.036	0.124	0.003	0.002
1977	0.012	0.037	0.128	0.006	0.023
1978	0.024	0.044	0.120	0.013	0.020
1979	0.020	0.078	0.095	0.021	0.044
1980	0.040	0.103	0.038	0.009	0.062
1981	0.079	0.183	0.250	0.028	0.165
1982	0.120	0.386	0.349	0.024	0.126
1983	0.063	0.300	0.472	0.041	0.090
1984	0.279	0.536	0.489	0.054	0.399
1985	0.239	0.374	0.672	0.054	0.160
1986	0.291	0.367	0.578	0.054	0.305
1987	0.321	0.591	0.556	0.073	0.436
1988	0.586	0.643	0.632	0.073	0.371
1989	0.483	0.728	0.379	0.115	0.545
1990	0.202	0.871	0.249	0.066	0.578
1991	0.289	0.735	0.339	0.115	0.246
1992	0.416	0.313	0.322	0.089	0.337
1993	0.299	0.276	0.184	0.070	0.302
Basic metals					
1972	0.042	0.239	0.054	0.132	0.265
1973	0.054	0.158	0.113	0.101	0.160
1974	0.065	0.141	0.096	0.080	0.141
1975	0.069	0.170	0.107	0.103	0.250
1976	0.060	0.143	0.081	0.093	0.204
1977	0.138	0.201	0.505	0.151	0.511
1978	0.060	0.240	0.113	0.081	0.546
1979	0.080	0.213	0.204	0.113	0.323
1980	0.058	0.205	0.324	0.084	0.183
1981	0.056	0.197	0.268	0.144	0.255
1982	0.080	0.231	0.167	0.115	0.142
1983	0.125	0.330	0.203	0.103	0.213
1984	0.131	0.285	0.170	0.073	0.200
1985	0.162	0.293	0.142	0.069	0.228
1986	0.168	0.381	0.192	0.141	0.202
1987	0.144	0.336	0.188	0.155	0.175
1988	0.140	0.336	0.153	0.284	0.204
1989	0.106	0.177	0.157	0.213	0.170
1990	0.064	0.136	0.149	0.114	0.275
1991	0.148	0.137	0.128	0.104	0.226
1992	0.156	0.165	0.254	0.151	0.368
1993	0.174	0.120	0.094	0.042	0.337

[1] Significance probability. Chi-square distribution with two degrees of freedom.

Table D.2. Testing for skewness of log-output and log-input variables[1]

Year	log(X)	log(K)	log(L)	log(E)	log(M)
Pulp and paper					
1972	0.071	0.056	0.182	0.297	0.031
1973	0.251	0.021	0.320	0.322	0.035
1974	0.036	0.034	0.161	0.317	0.002
1975	0.011	0.042	0.123	0.193	0.000
1976	0.030	0.023	0.223	0.262	0.000
1977	0.005	0.012	0.069	0.125	0.006
1978	0.012	0.014	0.041	0.130	0.005
1979	0.046	0.028	0.052	0.039	0.033
1980	0.059	0.033	0.013	0.114	0.055
1981	0.121	0.066	0.157	0.156	0.125
1982	0.143	0.194	0.368	0.428	0.045
1983	0.020	0.121	0.464	0.188	0.029
1984	0.288	0.265	0.719	0.341	0.213
1985	0.173	0.161	0.514	0.358	0.058
1986	0.311	0.159	0.506	0.475	0.132
1987	0.679	0.306	0.874	0.672	0.254
1988	0.552	0.353	0.887	0.709	0.192
1989	0.715	0.425	0.806	0.686	0.467
1990	0.633	0.653	0.632	0.804	0.398
1991	0.966	0.433	0.830	0.748	0.116
1992	0.849	0.325	0.428	0.972	0.647
1993	0.519	0.346	0.341	0.539	0.585
Basic metals					
1972	0.806	0.746	0.684	0.317	0.808
1973	0.958	0.626	0.890	0.152	0.800
1974	0.824	0.743	0.823	0.128	0.809
1975	0.877	0.759	0.989	0.208	0.112
1976	0.786	0.703	0.828	0.138	0.701
1977	0.487	0.954	0.564	0.280	0.297
1978	0.905	0.930	0.562	0.113	0.671
1979	0.345	0.737	0.953	0.305	0.133
1980	0.285	0.794	0.782	0.248	0.575
1981	0.137	0.605	0.856	0.343	0.602
1982	0.245	0.618	0.661	0.196	0.797
1983	0.359	0.641	0.563	0.174	0.646
1984	0.532	0.665	0.644	0.138	0.668
1985	0.225	0.582	0.942	0.104	0.610
1986	0.239	0.584	0.982	0.273	0.512
1987	0.258	0.455	0.788	0.253	0.447
1988	0.090	0.522	0.781	0.461	0.275
1989	0.073	0.787	0.841	0.268	0.393
1990	0.027	0.844	0.782	0.322	0.254
1991	0.190	0.914	0.730	0.369	0.501
1992	0.096	0.832	0.837	0.614	0.339
1993	0.445	0.374	0.434	0.208	0.888

[1] Two-tailed significance probability. Standard normal distribution.

Table D.3. Testing for excess kurtosis of log-output and log-input variables[1]

Year	log(X)	log(K)	log(L)	log(E)	log(M)
Pulp and paper					
1972	0.045	0.313	0.167	0.002	0.249
1973	0.009	0.489	0.115	0.001	0.302
1974	0.038	0.180	0.065	0.000	0.468
1975	0.181	0.171	0.180	0.002	0.649
1976	0.050	0.223	0.101	0.001	0.568
1977	0.341	0.566	0.372	0.005	0.872
1978	0.262	0.633	0.799	0.012	0.873
1979	0.049	0.599	0.334	0.063	0.190
1980	0.089	0.892	0.533	0.009	0.173
1981	0.102	0.918	0.379	0.023	0.263
1982	0.148	0.639	0.255	0.009	0.717
1983	0.708	0.916	0.326	0.031	0.838
1984	0.233	0.953	0.254	0.026	0.593
1985	0.315	0.938	0.544	0.025	0.773
1986	0.230	0.876	0.418	0.021	0.745
1987	0.147	0.963	0.284	0.024	0.550
1988	0.398	0.884	0.344	0.024	0.596
1989	0.250	0.987	0.170	0.041	0.408
1990	0.085	0.784	0.110	0.020	0.536
1991	0.115	0.971	0.146	0.040	0.565
1992	0.190	0.244	0.200	0.028	0.161
1993	0.157	0.194	0.115	0.026	0.148
Basic metals					
1972	0.012	0.097	0.017	0.081	0.107
1973	0.016	0.063	0.037	0.112	0.058
1974	0.020	0.051	0.031	0.098	0.049
1975	0.021	0.063	0.034	0.085	0.624
1976	0.018	0.053	0.026	0.111	0.082
1977	0.062	0.073	0.310	0.106	0.615
1978	0.018	0.091	0.045	0.112	0.310
1979	0.041	0.084	0.075	0.069	0.980
1980	0.033	0.078	0.140	0.057	0.079
1981	0.060	0.084	0.107	0.085	0.117
1982	0.055	0.101	0.065	0.103	0.050
1983	0.068	0.157	0.091	0.100	0.090
1984	0.055	0.128	0.068	0.081	0.081
1985	0.141	0.142	0.048	0.100	0.101
1986	0.140	0.202	0.069	0.100	0.096
1987	0.107	0.203	0.071	0.120	0.088
1988	0.304	0.183	0.055	0.160	0.159
1989	0.260	0.066	0.056	0.172	0.094
1990	0.444	0.047	0.054	0.067	0.257
1991	0.147	0.046	0.046	0.054	0.112
1992	0.330	0.059	0.101	0.060	0.298
1993	0.088	0.063	0.042	0.029	0.142

[1] Two-tailed significance probability. Standard normal distribution.

Appendix E. Microeconometric results

Table E.1. Estimates of parameters in the micro CD production functions

	Pulp and paper		Basic metals	
Parameter	Estimate	Standard error	Estimate	Standard error
$\overline{\alpha}^*$	-2.3021	0.2279	-3.1177	0.2702
κ	0.0065	0.0013	0.0214	0.0021
$\overline{\beta}_K$	0.2503	0.0344	0.1246	0.0472
$\overline{\beta}_L$	0.1717	0.0381	0.2749	0.0550
$\overline{\beta}_E$	0.0854	0.0169	0.2138	0.0374
$\overline{\beta}_M$	0.5666	0.0309	0.4928	0.0406
$\overline{\beta}$	1.0740	0.0287	1.1061	0.0324

Table E.2. The distribution of plant specific coefficients. Variances on the main diagonal and correlation coefficients below

Pulp and paper	α_i^*	β_{Ki}	β_{Li}	β_{Ei}	β_{Mi}
α_i^*	5.9336				
β_{Ki}	-0.4512	0.1147			
β_{Li}	-0.7274	-0.0559	0.1515		
β_{Ei}	0.3968	-0.4197	-0.3009	0.0232	
β_{Mi}	0.3851	-0.6029	-0.4262	0.1437	0.1053
Basic metals	α_i^*	β_{Ki}	β_{Li}	β_{Ei}	β_{Mi}
α_i^*	3.5973				
β_{Ki}	-0.0787	0.1604			
β_{Li}	-0.6846	-0.5503	0.1817		
β_{Ei}	0.3040	-0.6281	0.1366	0.1190	
β_{Mi}	0.1573	0.1092	-0.3720	-0.6122	0.1200

References

Antle, J.M. (1983): Testing the Stochastic Structure of Production: A Flexible Moment Based Approach. *Journal of Business & Economic Statistics* 1, 192-201.

Biørn, E., Lindquist, K.-G., and Skjerpen, T. (2000): Micro Data on Capital Inputs: Attempts to Reconcile Stock and Flow Information. Discussion Paper No. 268, Statistics Norway.

Biørn, E., Lindquist, K.-G., and Skjerpen, T. (2002): Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data. *Journal of Productivity Analysis* 18, 39-57.

Biørn, E., and Skjerpen, T. (2002): Aggregation and Aggregation Biases in Production Functions: A Panel Data Analysis of Translog Models. Discussion Paper No. 317, Statistics Norway.

Davidson, R., and MacKinnon, J.G. (1993): *Estimation and Inference in Econometrics.* New York: Oxford Economic Press.

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997): *Modelling Extremal Events for Insurance and Finance.* Berlin: Springer.

Evans, M., Hastings, N., and Peacock, B. (1993): *Statistical Distributions*, Second Edition. New York: Wiley.

Fortin, N.M. (1991): Fonctions de production et biais d'agrégation. *Annales d'Économie et de Statistique* 20/21, 41-68.

Greene, W.H. (2003): *Econometric Analysis*, Fifth edition. London: Prentice Hall.

Hall, B.H., and Cummins, C. (1999): *Time Series Processor Version 4.5.* Palo Alto, CA: TSP International.

Hildenbrand, W. (1998): How Relevant Are Specifications of Behavioral Relations on the Micro Level for Modelling the Time Path of Population Aggregates? *European Economic Review* 42, 437-458.

Jorgenson, D.W. (1995): *Productivity. Volume 2: International Comparisons of Economic Growth.* Cambridge, MA: MIT Press.

Klette, T.J., and Griliches, Z. (1996): The Inconsistency of Common Scale Estimators when Output Prices Are Unobserved and Endogenous. *Journal of Applied Econometrics* 11, 343-361.

Lamperti, J.W. (1996): *Probability. A Survey of the Mathematical Theory.* Second Edition. New York: Wiley.

Littell, R.C., Milliken, G.A., Stroup, W.W., and Wolfinger, R.D. (1996): *SAS System for Mixed Models.* Cary, NC: SAS Institute.

Lütkepohl, H. (1996): *Handbook of Matrices.* Chichester: Wiley.

Mas-Colell, A., Whinston, M.D., and Green, J.R. (1995): *Microeconomic Theory.* New York: Oxford University Press.

Magnus, J.R., and Neudecker, H. (1988): *Matrix Differential Calculus with Applications in Statistics and Econometrics.* Chichester: Wiley.

McCulloch, J.H. (1986): Simple Consistent Estimators of Stable Distribution Parameters. *Communications in Statistics – Simulation and Computation* 15, 1109-1136.

Statistics Norway (1995): *Energy Statistics 1995*. Official Statistics of Norway, NOS C 347.

Stoker, T.M. (1993): Empirical Approaches to the Problem of Aggregation Over Individuals. *Journal of Economic Literature* 31, 1827-1874.

Recent publications in the series Discussion Papers

251 T. Fæhn and E. Holmøy (1999): Welfare Effects of Trade Liberalisation in Distorted Economies. A Dynamic General Equilibrium Assessment for Norway

252 R. Aaberge (1999): Sampling Errors and Cross-Country Comparisons of Income Inequality

253 I. Svendsen (1999): Female labour participation rates in Norway – trends and cycles

254 A. Langørgen and R. Aaberge: A Structural Approach for Measuring Fiscal Disparities

255 B. Halvorsen and B.M. Larsen (1999): Changes in the Pattern of Household Electricity Demand over Time

256 P. Boug (1999): The Demand for Labour and the Lucas Critique. Evidence from Norwegian Manufacturing

257 M. Rege (1999): Social Norms and Private Provision of Public Goods: Endogenous Peer Groups

258 L. Lindholt (1999): Beyond Kyoto: CO_2 permit prices and the markets for fossil fuels

259 R. Bjørnstad and R. Nymoen (1999): Wage and Profitability: Norwegian Manufacturing 1967-1998

260 T.O. Thoresen and K.O. Aarbu (1999): Income Responses to Tax Changes – Evidence from the Norwegian Tax Reform

261 B. Bye and K. Nyborg (1999): The Welfare Effects of Carbon Policies: Grandfathered Quotas versus Differentiated Taxes

262 T. Kornstad and T.O. Thoresen (1999): Means-testing the Child Benefit

263 M. Rønsen and M. Sundström (1999): Public Policies and the Employment Dynamics among new Mothers – A Comparison of Finland, Norway and Sweden

264 J.K. Dagsvik (2000): Multinomial Choice and Selectivity

265 Y. Li (2000): Modeling the Choice of Working when the Set of Job Opportunities is Latent

266 E. Holmøy and T. Hægeland (2000): Aggregate Productivity and Heterogeneous Firms

267 S. Kverndokk, L. Lindholt and K.E. Rosendahl (2000): Stabilisation of CO_2 concentrations: Mitigation scenarios using the Petro model

268 E. Biørn, K-G. Lindquist and T. Skjerpen (2000): Micro Data On Capital Inputs: Attempts to Reconcile Stock and Flow Information

269 I. Aslaksen and C. Koren (2000): Child Care in the Welfare State. A critique of the Rosen model

270 R. Bjørnstad (2000): The Effect of Skill Mismatch on Wages in a small open Economy with Centralized Wage Setting: The Norwegian Case

271 R. Aaberge (2000): Ranking Intersecting Lorenz Curves

272 J.E. Roemer, R. Aaberge , U. Colombino, J, Fritzell, S.P. Jenkins, I. Marx, M. Page, E. Pommer, J. Ruiz-Castillo, M. Jesus SanSegundo, T. Tranaes, G.G.Wagner and I. Zubiri (2000): To what Extent do Fiscal Regimes Equalize Opportunities for Income Acquisition Among citizens?

273 I. Thomsen and L.-C. Zhang (2000): The Effect of Using Administrative Registers in Economic Short Term Statistics: The Norwegian Labour Force Survey as a Case Study

274 I. Thomsen, L.-C. Zhang and J. Sexton (2000): Markov Chain Generated Profile Likelihood Inference under Generalized Proportional to Size Non-ignorable Non-response

275 A. Bruvoll and H. Medin (2000): Factoring the environmental Kuznets curve. Evidence from Norway

276 I. Aslaksen, T. Wennemo and R. Aaberge (2000): "Birds of a feather flock together". The Impact of Choice of Spouse on Family Labor Income Inequality

277 I. Aslaksen and K.A. Brekke (2000): Valuation of Social Capital and Environmental Externalities

278 H. Dale-Olsen and D. Rønningen (2000): The Importance of Definitions of Data and Observation Frequencies for Job and Worker Flows - Norwegian Experiences 1996-1997

279 K. Nyborg and M. Rege (2000): The Evolution of Considerate Smoking Behavior

280 M. Søberg (2000): Imperfect competition, sequential auctions, and emissions trading: An experimental evaluation

281 L. Lindholt (2000): On Natural Resource Rent and the Wealth of a Nation. A Study Based on National Accounts in Norway 1930-95

282 M. Rege (2000): Networking Strategy: Cooperate Today in Order to Meet a Cooperator Tomorrow

283 P. Boug, Å. Cappelen and A.R. Swensen (2000): Expectations in Export Price Formation: Tests using Cointegrated VAR Models

284 E. Fjærli and R. Aaberge (2000): Tax Reforms, Dividend Policy and Trends in Income Inequality: Empirical Evidence based on Norwegian Data

285 L.-C. Zhang (2000): On dispersion preserving estimation of the mean of a binary variable from small areas

286 F.R. Aune, T. Bye and T.A. Johnsen (2000): Gas power generation in Norway: Good or bad for the climate? Revised version

287 A. Benedictow (2000): An Econometric Analysis of Exports of Metals: Product Differentiation and Limited Output Capacity

288 A. Langørgen (2000): Revealed Standards for Distributing Public Home-Care on Clients

289 T. Skjerpen and A.R. Swensen (2000): Testing for long-run homogeneity in the Linear Almost Ideal Demand System. An application on Norwegian quarterly data for non-durables

290 K.A. Brekke, S. Kverndokk and K. Nyborg (2000): An Economic Model of Moral Motivation

291 A. Raknerud and R. Golombek: Exit Dynamics with Rational Expectations

292 E. Biørn, K-G. Lindquist and T. Skjerpen (2000): Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data

293 K-G. Lindquist and T. Skjerpen (2000): Explaining the change in skill structure of labour demand in Norwegian manufacturing

294 K. R. Wangen and E. Biørn (2001): Individual Hetero-geneity and Price Responses in Tobacco Consumption: A Two-Commodity Analysis of Unbalanced Panel Data

295 A. Raknerud (2001): A State Space Approach for Estimating VAR Models for Panel Data with Latent Dynamic Components

296 J.T. Lind (2001): Tout est au mieux dans ce meilleur des ménages possibles. The Pangloss critique of equivalence scales

297 J.F. Bjørnstad and D.E. Sommervoll (2001): Modeling Binary Panel Data with Nonresponse

298 Taran Fæhn and Erling Holmøy (2001): Trade Liberalisation and Effects on Pollutive Emissions and Waste. A General Equilibrium Assessment for Norway

299 J.K. Dagsvik (2001): Compensated Variation in Random Utility Models

300 K. Nyborg and M. Rege (2001): Does Public Policy Crowd Out Private Contributions to Public Goods?

301 T. Hægeland (2001): Experience and Schooling: Substitutes or Complements

302 T. Hægeland (2001): Changing Returns to Education Across Cohorts. Selection, School System or Skills Obsolescence?

303 R. Bjørnstad: (2001): Learned Helplessness, Discouraged Workers, and Multiple Unemployment Equilibria in a Search Model

304 K. G. Salvanes and S. E. Førre (2001): Job Creation, Heterogeneous Workers and Technical Change: Matched Worker/Plant Data Evidence from Norway

305 E. R. Larsen (2001): Revealing Demand for Nature Experience Using Purchase Data of Equipment and Lodging

306 B. Bye and T. Åvitsland (2001): The welfare effects of housing taxation in a distorted economy: A general equilibrium analysis

307 R. Aaberge, U. Colombino and J.E. Roemer (2001): Equality of Opportunity versus Equality of Outcome in Analysing Optimal Income Taxation: Empirical Evidence based on Italian Data

308 T. Kornstad (2001): Are Predicted Lifetime Consumption Profiles Robust with respect to Model Specifications?

309 H. Hungnes (2001): Estimating and Restricting Growth Rates and Cointegration Means. With Applications to Consumption and Money Demand

310 M. Rege and K. Telle (2001): An Experimental Investigation of Social Norms

311 L.C. Zhang (2001): A method of weighting adjustment for survey data subject to nonignorable nonresponse

312 K. R. Wangen and E. Biørn (2001): Prevalence and substitution effects in tobacco consumption. A discrete choice analysis of panel data

313 G.H. Bjertnær (2001): Optimal Combinations of Income Tax and Subsidies for Education

314 K. E. Rosendahl (2002): Cost-effective environmental policy: Implications of induced technological change

315 T. Kornstad and T.O. Thoresen (2002): A Discrete Choice Model for Labor Supply and Child Care

316 A. Bruvoll and K. Nyborg (2002): On the value of households' recycling efforts

317 E. Biørn and T. Skjerpen (2002): Aggregation and Aggregation Biases in Production Functions: A Panel Data Analysis of Translog Models

318 Ø. Døhl (2002): Energy Flexibility and Technological Progress with Multioutput Production. Application on Norwegian Pulp and Paper Industries

319 R. Aaberge (2002): Characterization and Measurement of Duration Dependence in Hazard Rate Models

320 T. J. Klette and A. Raknerud (2002): How and why do Firms differ?

321 J. Aasness and E. Røed Larsen (2002): Distributional and Environmental Effects of Taxes on Transportation

322 E. Røed Larsen (2002): The Political Economy of Global Warming: From Data to Decisions

323 E. Røed Larsen (2002): Searching for Basic Consumption Patterns: Is the Engel Elasticity of Housing Unity?

324 E. Røed Larsen (2002): Estimating Latent Total Consumption in a Household.

325 E. Røed Larsen (2002): Consumption Inequality in Norway in the 80s and 90s.

326 H.C. Bjørnland and H. Hungnes (2002): Fundamental determinants of the long run real exchange rate:The case of Norway.

327 M. Søberg (2002): A laboratory stress-test of bid, double and offer auctions.

328 M. Søberg (2002): Voting rules and endogenous trading institutions: An experimental study.

329 M. Søberg (2002): The Duhem-Quine thesis and experimental economics: A reinterpretation.

330 A. Raknerud (2002): Identification, Estimation and Testing in Panel Data Models with Attrition: The Role of the Missing at Random Assumption

331 M.W. Arneberg, J.K. Dagsvik and Z. Jia (2002): Labor Market Modeling Recognizing Latent Job Attributes and Opportunity Constraints. An Empirical Analysis of Labor Market Behavior of Eritrean Women

332 M. Greaker (2002): Eco-labels, Production Related Externalities and Trade

333 J. T. Lind (2002): Small continuous surveys and the Kalman filter

334 B. Halvorsen and T. Willumsen (2002): Willingness to Pay for Dental Fear Treatment. Is Supplying Fear Treatment Social Beneficial?

335 T. O. Thoresen (2002): Reduced Tax Progressivity in Norway in the Nineties. The Effect from Tax Changes

336 M. Søberg (2002): Price formation in monopolistic markets with endogenous diffusion of trading information: An experimental approach

337 A. Bruvoll og B.M. Larsen (2002): Greenhouse gas emissions in Norway. Do carbon taxes work?

338 B. Halvorsen and R. Nesbakken (2002): A conflict of interests in electricity taxation? A micro econometric analysis of household behaviour

339 R. Aaberge and A. Langørgen (2003): Measuring the Benefits from Public Services: The Effects of Local Government Spending on the Distribution of Income in Norway

340 H. C. Bjørnland and H. Hungnes (2003): The importance of interest rates for forecasting the exchange rate

341 A. Bruvoll, T.Fæhn and Birger Strøm (2003): Quantifying Central Hypotheses on Environmental Kuznets Curves for a Rich Economy: A Computable General Equilibrium Study

342 E. Biørn, T. Skjerpen and K.R. Wangen (2003): Parametric Aggregation of Random Coefficient Cobb-Douglas Production Functions: Evidence from Manufacturing Industries