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Optimal indirect taxation: A review of theoretical and empirical results

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Optimal indirect taxation: A review of theoretical and empirical results*

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Abstract

This paper offers a review of the theoretical and empirical contributions in the literature of optimal indirect taxation. The first part focuses on the theoretical contributions and discusses the role of indirect taxation in the presence of a proportional, a linear or an unrestricted (non-linear) direct tax respectively. The second part is occupied with the empirical contributions in the literature. I discuss the information requirements needed for empirical studies. The tendency to disregard the need for global regularity concerning functional forms is a possible weakness of parts of the literature.

1 Introduction

This paper reviews the theoretical and empirical studies of optimal indirect taxation. Indirect taxes (commodity taxes) can be defined as taxes levied on supply of and demand for different goods. They can be general in the sense of relating to every good (e.g. VAT) or they can be specific in the sense of relating to certain types of goods (e.g. excise duties on alcohol). They can be levied as a percent of the price (ad valorem tax) or as a unit (specific) tax. Indirect taxes differ from direct taxes in the sense that the latter are levied on a physical or juristic person. In this paper we consider proportional indirect taxes in combination with a proportional, linear or unrestricted (non-linear) direct tax (income tax).

The purposes of indirect taxation can be listed as follows. First, indirect taxation can act as a purely fiscal instrument for raising revenue for the government. Second, it can act as a tool for internalizing costs (or benefits) not reflected in market price (externalities) or as a device for correcting

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individuals preferences (merit goods). Third, it can be a tool for redistributing resources (equity concerns). In general all these purposes will prevail simultaneously. We would like to know what kind of indirect tax structure that would be optimal in the sense of generating the highest welfare.

Ramsey's 'A contribution to the theory of taxation' (1927) was the first attempt to provide an analytical answer. The paper started what could be called the 'Ramsey tradition'. It assumes the existence of certain constraints on the tools available to the government, namely that all the taxes available are restricted to be linear and possibly depend on household characteristics. Based on these assumptions one may proceed to derive rules for optimality. Ramsey studied a one-consumer economy in which direct taxation was not allowed. Others extended the model to the many-consumer case (Diamond and Mirrlees, 1971; Diamond, 1975), with the possibility of linear income tax (Deaton, 1979a; Deaton and Stern, 1986), and with externalities (Sandmo, 1975). The model was subsequently interpreted and studied by several other scholars (Baumol and Bradford, 1970; Besley and Jewitt, 1995; Corlett and Hague, 1953; Deaton, 1979b; Deaton, 1981; Dixit, 1970, 1975, 1979; Feldstein, 1972; Lerner, 1970; Mirrlees,1974; Munk, 1977, 1980; Sadka, 1977; Samuelson, 1982; Sandmo,1974, 1976; Stiglitz and Dasgupta, 1971).

James Mirrlees gave birth to what we can call the 'Mirrlees tradition' (Mirrlees, 1971, 1976). Here the problem is not the lack of tools available, but asymmetric information since the planner cannot discern each individuals's typology. Mirrlees (Mirrlees, 1976) studied originally non-linearity in both direct and indirect taxes, but later studies have focused on a mix where we are allowed to employ a non-linear income tax along with proportional indirect taxes (Atkinson and Stiglitz, 1976; Stiglitz, 1982). As with the Ramsey tradition, several extensions exist, such as allowing for heterogeneous preferences (Saez, 2002), relaxing production efficiency (Naito, 1999), and incorporating externalities (Pirttilä and Tuomala, 1997).

I review both traditions to establish what the literature tell us about the properties of an optimal designed indirect tax system and to clarify assumptions underlying each result. While a review of theory might appeal primarily to a theoretically minded economist, it should be of no less concern to an empirical orientated economist trying to calculate optimal taxes. This is because certain model specifications completely dictate the entire structure of the indirect taxes a priori. A clear understanding of the theoretical results is therefore of great importance (Deaton, 1981).

The empirical contributions are few in comparison with the theoretical; they all belong to the the Ramsey tradition and operate under rather limiting and restrictive frameworks, possibly because the informational requirements are large for computing optimal taxes.

The paper is organized as follows. Section 2 presents and discusses the theory of optimal indirect taxation. First we concentrate on the Ramsey tradition, discussing both the one-consumer economy and the many-consumer economy. Next, advancing to the Mirrlees tradition we introduce the possibility of non-linear income tax, before discussing the impact of relaxing production efficiency. Finally we look at externalities, merit goods and cross-border shopping under the optimal tax framework.

Section 3 is devoted to the empirical contributions. Here we clarify what is needed for computing and implementing optimal taxes, and then present and discuss the empirical contributions. Section 4 sums up and conclude.

2 The optimal tax problem

We consider a perfectly competitive economy with N goods, H households, F producers and a planner. Household h maximizes its utility function u_h with respect to labor supply (L_h) and commodity demand $(x_{h1}, ..., x_{hN})$ conditioned on an externality generating commodity j and subject to a budget constraint[1][2]

$$\max_{\mathbf{x},L} u_h(x_{h1}, ..., x_{hN}, L_h; X_j) \tag{1}$$

$$s.t. \sum_i p_i x_{hi} = w_h L_h + \pi_h - T(w_h L_h, \pi_h, \mathbf{z}_h),$$

where w_h is the wage rate facing household h, $\pi_h = \sum_f \omega_{fh} \pi_f$ is the profit income to household h where ω_{fh} is household h owner share in firm f and π_f is the profit of firm f. $T(.)$ is the direct tax function which in general can depend on earnings $(w_h L_h)$, profit income and a vector containing household characteristics $(\mathbf{z}_h)$ (ex: number of children and adults). X_j is aggregate demand for commodity j representing an externality effect. $(p_1, .., p_N)$ are the consumer prices. These are determined by the producer prices $(q_{1,...}q_N)$ and the indirect tax system in the following way

$$p_i = (q_i + c_i)(1 + b_i), \tag{2}$$

where c_i is a unit tax and b_i an ad valorem tax. Given (2), it follows that the commodity taxes are proportional in consumption and expenditure measured by producer plus unit tax prices. Note, on the other hand, that the direct taxes be non-linear in earnings, household characteristics and profit. Thanks to this non-linearity the consumer may face a non-convex budget set.

From the problem in (1) we can derive indirect utility functions (v_h) depending on consumer prices, wage rate, earnings, disposable income $(w_h L_h -$

[1] Maximizing utility taking X_j as given means that the household's own consumption of the good comprises only a fraction of total demand.

[2] There must be some sort of agreement between the household members as it is the household that maximizes.

$T(w_h L_h)$) and household characteristics, namely the following

$$v_h(p_1, ...p_N, w_h, Y_h, M_h, \mathbf{z}_h, X_j), \tag{3}$$

where $Y_h = w_h L_h$ and $M_h = Y_h - T(Y_h, \pi_h, \mathbf{z}_h,)$.The indirect utility functions are here conditional in the sense of conditioning on earnings.

Each producer in the economy maximizes the profit subject to a production constraint :

$$\max_{\mathbf{x},L} \sum_i q_i x_{if} - \sum_h w_h L_{fh} \tag{4}$$

$$s.t. \;\; F_f \left(x_1, ..., x_{n,} L_1, ..., L_h\right) = 0.$$

We can now derive the profit for firm f as a function of the wage rates and product prices:

$$\pi_f(q_1, ..., q_N, w_1, ..., w_H) \tag{5}$$

What the planner needs to do in the economy is maximize the welfare given his need for tax revenue.[3] He does this by designing a tax scheme that determines the price structure facing the households and producers (i.e., by using indirect taxation) and assigns to each household a specific combination of earnings and disposable incomes (i.e., by using direct taxation). We can formulate the maximization problem as

$$\max_{\mathbf{p},\mathbf{q},\mathbf{Y},\mathbf{M}} W(v_1\left(\mathbf{p}, w_1, Y_1, M_{1,}\mathbf{z}_1, X_j\right), .., v_H(\mathbf{p}, w_H, Y_H, M_H, \mathbf{z}_H, X_j))$$

$$s.t. \;\; \sum_i X_i(q_i + c_i)b_i + \sum_i X_i c_i + \sum_h T\left(Y_h, \pi_h, \mathbf{z}_h\right) = R \tag{6}$$

$$v_k(\mathbf{p}, w_k, Y_k, M_k, \mathbf{z}_h, X_j) \geq v_k(\mathbf{p}, w_k, Y_h, M_h, \mathbf{z}_h, X_j) \quad \text{for all } h \neq k,$$

where $W(.)$ is the welfare function and X_i is aggregate private demand for good i. Revenue is the problem's first constraint. It simply states that the need for revenue (R) must be equal to the sum of tax revenue from indirect taxation, $\sum X_i(q_i + c_i)b_i + \sum X_i c_i$, and direct taxation $\sum T\left(Y_h, \pi_h, \mathbf{z}_h\right)$. The next constraint is associated with the possibility of non-linear direct taxation. As mentioned, the consumer could in this case face a non-convex budget set. This fact makes it necessary to include an incentive compatibility constraint to ensure that the combination of earnings and disposable income assigned for a specific individual is the combination he actually choose.[4]

[3] The planner will use the tax revenue to finance a given demand of different commodities and labor used for an undefined purpose. This interpretation ensures an economy wide balance of supply and demand.

[4] At first, it might have seemed strange to include earnings as well as the wage rate in (3). Since wage rate (together with consumer prices) determines labor supply and thereby

The first constraint in problem (6) can be written as

$$\sum_i t_i X_i + \sum_h T\left(Y_h, \pi_h, \mathbf{z}_h\right) = R, \tag{7}$$

where

$$t_i = q_i b_i + b_i c_i + c_i = b_i(q_i + c_i) + c_i. \tag{8}$$

Here t_i is the total tax on commodity i when measured as a unit tax. With perfect competition we are indifferent to how it is implemented in the sense of which combination of b and c we choose.[5] From (2) and (8) we have that $t_i = p_i - q_i$, such that the maximization problem can finally be written as

$$\max_{\mathbf{p},\mathbf{q},\mathbf{Y},\mathbf{M}} W(v_1\left(\mathbf{p}, w_1, Y_1, M_1, \mathbf{z}_1, X_j\right), .., v_H(\mathbf{p}, w_H, Y_H, M_H, \mathbf{z}_H, X_j) \tag{9}$$

$$s.t. \sum_i (p_i - q_i) X_i + \sum_h T\left(Y, \pi_h, \mathbf{z}_h\right) = R$$

$$v_k(\mathbf{p}, w_k, Y_k, M_k, \mathbf{z}_k, X_j) \geq v_k(\mathbf{p}, w_k, Y_h, M_h, \mathbf{z}_k, X_j) \text{ for all } h \neq k.$$

In the following we shall study the problem given in (9) under different specifications of the direct tax function $T(.)$.

2.1 Indirect taxation in the presence of linear - or proportional income tax: the Ramsey tradition

Under the Ramsey tradition the direct tax function can be specified as follows

$$T(Y, \pi) = aY + \tau\pi \tag{10}$$

or

$$T(Y, \pi) = m + aY + \tau\pi, \tag{11}$$

where τ is the profit tax rate. (10) represents a proportional income tax, whereas (11) defines a linear income tax. In (11), m may depend on household characteristics $\mathbf{z}$. A possible specification is

$$m_h = m_o + m_1 z_{h1} + m_2 z_{h2}, \tag{12}$$

where z_{h1} = number of adults in the household h, and z_{h2} = number of children in the household h. In this case (11) becomes

earnings, this could seem superflous. This formulation is necessary because designing a direct tax system is for the planner to choose combinations of earnings and disposable income to offer the households. In addition the direct tax system must, in general, fulfill the incentive compatability constraint. Otherwise, the tax system would not be possible to implement.

[5] This does not apply under conditions of imperfect competition. For a review, see Keen (1998).

$$T(Y, \pi, \mathbf{z}_h) = m_h + aY + \tau\pi. \tag{13}$$

In the following we assume that m is the same for all households, i.e. as in (11), unless stated to the contrary.

All these specifications make incentive compatibility constraint superfluous, since the households will only be facing convex budgets sets. The direct tax structure can be described by two parameters only in the former case, namely a and τ, and by three parameters in the latter case, namely a, τ and m.[6] We can therefore write the indirect utility function as

$$v_h\left(p_1, .., p_N, w_h, a, \tau, X_j\right) \tag{14}$$

for the proportional case, and

$$v_h\left(p_1, .., p_N, w_h, a, m, \tau, X_j\right) \tag{15}$$

for the linear case. The demand functions will consist of the same arguments in each case. The supply function will have as arguments all the producer prices. Notice that if we assume $\pi = 0$, then the demand functions will be homogenous of degree zero in consumer prices and $1 - a$, and in consumer prices and wage rate. Likewise, the supply functions for each firm will be homogenous of degree zero in all producer prices. If we then choose the wage rate as the fixed producer price we need to fix one consumer price or tax rate a for there to be a unique solution. This is purely a normalization and will not affect the quantity outcomes. [7]

In the case of decreasing returns to scale and profit, producer prices affect the consumer and demand by changing the profit. Multiplying all producer prices and consumer prices by a constant will therefore not alter the equilibrium, and we won't have to fix more than one price, for instance the wage rate. If we fix more, it would be to impose real restrictions on the problem.

2.1.1 When only efficiency matters

We start by focusing on optimal indirect taxation within the context of a proportional income tax and under the assumption that only efficiency aspect matters.[8] We consider a one-consumer economy or, assuming perfect

[6] In other words, the combinations of earnings and disposable income could completely be described by these parameters.

[7] In case (11) increasing all prices and $1 - a$ by a factor k will not alter the equilibrium because m then also has to increase by k for the budget and revenue constraint to hold.

[8] If we were to allow for linear direct tax in an economy without distributional concerns, we would reach a very trivial solution to our problem. We should then collect all the tax revenue by a negative transfer to be reflected by the parameter m. This would leave us with a pure poll tax generating no efficiency loss.

aggregation is possible, a representative consumer in a many-consumer economy with no externalities. We assume constant returns to scale and hence that no profit exists, further that we have constant producer prices. Let us set up the analytical formulation of the maximization problem

$$\max_p V(p_1, .., p_N, w, a)$$

$$\text{s.t.} \tag{16}$$

$$\sum X_i(p_i - q_i) = R,$$

where V is the indirect utility of the consumer. Under the assumption of constant producer prices we need to fix one consumer price or the income tax rate to get a unique solution. From our point of view it would be practical to fix the income tax rate a and ask how the indirect tax system should be designed given the income tax.[9]

Maximizing (16) with respect to p_i gives the following first order conditions

$$\frac{\partial V}{\partial p_k} + \lambda(X_k + \sum t_i \frac{\partial X_i}{\partial p_k} + a\frac{\partial Y}{\partial p_k}) = 0, \tag{17}$$

where λ is the Lagrange multiplier. The two first terms inside the parenthesis is increase in indirect tax revenue from the change in consumer price, and the third term is the increase in direct tax revenue, income tax revenue. This can further be written as

$$\frac{\frac{\partial V}{\partial p_k}}{X_k + \sum t_i \frac{\partial X_i}{\partial p_k} + a\frac{\partial Y}{\partial p_k}} = -\lambda, \tag{18}$$

which states that in optimum the reduction in utility per unit increase in tax revenue (from indirect and direct taxation) must be the same for all commodities. This must obviously hold for every optimal tax problem at optimum.

Let I be an exogenous income to the household. Availing ourselves of the Slutsky equation

$$\frac{\partial X_i}{\partial p_k} = S_{ik} - X_k \frac{\partial X_i}{\partial I}, \tag{19}$$

and its symmetry property together with Roys identity, this first order condition can be written

$$\frac{\sum t_i S_{ik}}{X_k} = \frac{\gamma - \lambda}{\lambda} - \frac{a\frac{\partial Y}{\partial p_k}}{X_k}, \tag{20}$$

[9] On fixing the tax rate in this case we are implicitly fixing one consumer price, namely the after tax wage rate.

where $\gamma = \alpha + \lambda \sum t_i \frac{\partial X_i}{\partial I}$ and $\alpha = \frac{\partial V}{\partial I}$. This equation is a modified version of the classical Ramsey equation and reduces to the original one when setting $a = 0$. Again, using the Slutsky equation and its properties along with the fact that $\frac{\partial Y}{\partial p_j} = w\frac{\partial L}{\partial p_j}$ when assuming constant producer prices, we can rewrite this equation as

$$\frac{\sum t_i S_{ik}}{X_k} = \frac{\gamma^D - \lambda}{\lambda} - aw\frac{S_{kw}}{X_k} \Longrightarrow \tag{21}$$

$$\frac{\sum t_i S_{ik} + awS_{kw}}{X_k} = \frac{\gamma^D - \lambda}{\lambda}, \tag{22}$$

where $\gamma^D = \alpha + \lambda \left(\sum t_i \frac{\partial X_i}{\partial I} + aw\frac{\partial L}{\partial I} \right)$. As (22) states, the relative reduction in compensated demand following a small intensification of the whole tax system should be the same for all goods at optimum. Since the tax system in this case includes the direct tax, a small intensification of the indirect tax structure only, should not in general decrease the relative compensated demand equally for all goods, unlike the classical result which sets $a = 0$.

Firstly, it is the impact on real variables, not prices, that counts when designing an optimal tax regime. Prices play the role as instruments for redistributing resources efficiently. Nevertheless, we might feel tempted to recommend as the solution to our tax problem: 'increase all the prices in the same proportions', which, we could argue, gives us no efficiency loss. As Sandmo (1975) explains, this obviously is wrong. We would only be subsidizing the supply from consumers (such as labor) and taxing the demand such that in sum it would not raise any revenue at all. Translated to our case it would mean introducing a earning subsidy equal to the tax revenue from indirect taxation. By introducing the so-called discouraging index, Mirrless (1976) shows his awareness of this point. This index shows the degree to which each commodity's demand should be distorted.

Secondly, the only thing that matters is the compensated demand. When we know that substitution effects are solely responsible for the change in compensated demand, this might not seem so surprising. It is the substitution effect that matters for efficiency loss, not income effects. The income effect is purely a transfer effect, i.e., the consumer loses one dollar and the planner gains one. If prices are set optimally the planner could never do better than this. The substitution effect has to do with changes in behavior caused by tax wedges that generates a non-Pareto optimal price structure. The consumer could then lose 1.2 dollar and the planner still gain one. We would like to reduce the consumer's loss and get as close to one dollar as possible. To do this we try to minimize these substitution effects.

For more insight on this, we can turn to Auerbach (1985) where we will find a clarifying discussion. The fundamental problem, says Auerbach, is that we can only tax the sales of a good. Consumption and sale only coincide when the consumer has no stock of a the good. Since the consumer always has a stock of leisure available, sale and consumption will never coincide for this good. If we could tax the stock of leisure, we could have designed a tax system that yielded no efficiency loss. In practice it would mean reverting to slavery.

This rule does not tell us much about the magnitude of the individual tax rates in optimum and the properties of the tax structure. We need special cases to get some answers on this.[10] Let us look at the issue of uniformity.

Uniformity In the case of uniform structure the tax rates t_i can be written as $t_i = p_i d$, where d is constant for all i. Substituting into equation (22) and using the fact that $\sum p_i S_{ik} = -wS_{kw}$ we get

$$\frac{wS_{kw}}{X_k} = \frac{\gamma^D - \lambda}{\lambda} \frac{1}{a-d}. \tag{23}$$

The term on the left side is the compensated elasticity of demand for commodity k with respect to the wage rate. Since the right hand side is a constant in the case of uniform taxation, we can conclude that equal compensated elasticities gives a uniform structure. In fact Sadka (1977) proves this to be both a necessary and sufficient condition for uniform taxation. A preference structure that satisfies this is the case where the direct utility can be written as $U = U(u(x), L)$ and where $u(.)$ is homogenous of degree one. The U function is weakly separable in commodities and labor with the result that demand for commodity i can be written as function of commodity prices and disposable income. This implies that

$$S_{kw} = \frac{\partial X_k}{\partial M} \cdot \frac{\partial M}{\partial w}_{u=const.}, \tag{24}$$

because of the fact that the only channel through which a wage increase could affect the commodity demand is income. Multiplying both sides with $\frac{w}{X_k}$ gives

$$\frac{wS_{kw}}{X_k} = El_M X_k \cdot El_w M|_{u=const.} . \tag{25}$$

As can be seen, the left side is independent of k if the Engel elasticity is equal for every commodity, i.e. Engel elasticities equal to one. If we let $u(x)$ be a Cobb-Douglas function, the Engel elasticities are all equal to one and uniform structure is optimal.

To gain some intuition for this realize that a uniform commodity tax structure could always be implemented as a proportional income tax, but a differentiated commodity tax structure could not always be mimicked by the income tax alone. For a differentiated structure to be optimal it must add

[10]Note that we could have derived the results above without assuming constant producer prices. When we explore the indirect tax structure below, the assumption of constant producer prices is crucial.

something more than the proportional income tax alone can accomplish, i.e., it must broaden the range of tax instruments in some sense. In the one-consumer case, the consumer's consumption pattern must necessarily change when income and leisure change. We can then let the consumer pay more or less tax measured as share of income for different income levels. From consumer theory we know that with linear Engel curves through the origin a consumer will have equal consumption pattern for every income level, i.e. the Engel elasticities will all be equal to one (see Deaton and Muellbauer (1980)). This might lead us to conclude that we already have a condition for uniformity in the one-consumer case, but we must be careful. As Sandmo (1975) points out, we must add an assumption of weak separability between leisure (the numeraire good) and the commodities. Without this assumption the consumption pattern would still change when income changes due to the change in wage and consumption of leisure.

Furthermore, it follows from Deaton (1981) that if we can implicitly define the utility function in the form[11]

$$g(\vartheta(X,u),L,u)=1, \tag{26}$$

where $\vartheta()$ is homogenous of degree one in its arguments, then a uniform structure will be optimal. In this case the commodity budget shares can be written as a function of the utility and commodity prices alone. Hence, if we compensate for an introduction of income tax (keeping utility fixed), this will effect a proportional change in compensated demand. This kind of separability in cost function is known as implicit or quasi separability. Elaborating further, Besley and Jewitt (1995) find that if the utility function could be written on the implicit form

$$f(\vartheta(X,L,u),L,u)=1, \tag{27}$$

it would constitute a sufficient condition for uniform structure. They show that it contains Deaton's result when assuming that $\vartheta(.)$ is homogenous of degree one, and contains a utility function which Mirrlees (1984) has shown to give a uniform structure, namely the function

$$U=H(\phi(\frac{X}{M}),M), \tag{28}$$

when assuming that $\vartheta(.)$ is independent of U.

In the case of perfectly inelastic labor supply a uniform indirect tax structure will always be optimal. The taxes will therefore act as an poll tax,

[11] This gives separability in the cost function and can equivalently be written as

$$C(U,w,p)=C^*(U,w,\vartheta(U,p)),$$

which is what Deaton (1981) actually uses (see Besley and Jewitt, 1995, pp. 81-82).

reducing the consumers' income but not inducing any substitution between labor and commodity demand. Usually, if we have a good that is perfectly inelastic, this good should bear all taxes or, equivalently, all the other goods should face a uniform tax structure.

Non uniformity Baumoul and Bradford (1970) derived the so-called inverse elasticity rule by assuming there are no cross price effects between the commodities in the economy, i.e. the demand for a certain good depends only on the price of that good and the price of leisure, namely the wage. Substituting $\partial X_i/\partial p_k = 0$ for $i \neq k$, our first order condition reduces to

$$\frac{t_k}{q_k + t_k} = \left[\frac{\alpha - \lambda}{\lambda}\right] \frac{1}{e_k} - \frac{aw}{p_k} \frac{\partial L}{\partial X_k}, \tag{29}$$

where e_k is the price elasticity of demand for good k. The equation is a slightly modified version of the inverse elasticity rule and reduces to it when $a = 0$. It states that the proportional tax rate should be inversely related to the price elasticity of the good taxed and smaller the more complementary the good is with labor. Within this framework the relationship between the price elasticity and the $\frac{\partial L}{\partial X_k}$ must be a positive monotone relation, so it continues to hold that we should tax the goods with low price elasticity higher.[12] Since necessities typically will have low elasticities of demand, the rule calls for a regressive commodity tax structure. Uniform tax structure will prevail when all price elasticities are equal.

Within a three-good setting (leisure and two commodities) Corlett and Hague (1953) try to determine which kind of commodities should be taxed as a supplement to an already existing optimal income tax. Their analysis relies on a marginal reform approach which considers the welfare change of introducing differentiated commodity tax structure when taking as a starting point a uniform system, alternatively interpreted as an existing proportional income tax. They find that the commodity which was a stronger complement (in Hicks sense) to leisure should bear a higher tax. This is a marginal analysis, however, not a global one. Still, the result also holds within an optimal design framework (See Sandmo (1976)). Nevertheless one should stress that we cannot say anything certain about the many commodity case.

Deaton (1981) shows that weak separability between commodities and leisure gives a regressive indirect tax structure. The definition of a regressive structure in a one-consumer economy is a structure which taxes luxuries less in optimum.

[12]Moreover, when $\mathrm{e}_k > -1$ we have $\frac{\partial L}{\partial X_k} > 0$. Then $\frac{\partial L}{\partial X_k} = 0$ when $e_k = -1$. And when $e_k < -1$ then $\frac{\partial L}{\partial X_k} < 0$ then $e_k < -1$. All this could be seen from inspecting the consumers budgets constraint.

Decreasing returns to scale and profit In the above, the production technology had the property of constant returns to scale in addition to constant producer prices. Under the assumption of decreasing returns to scale, producer prices will affect the profit and eventually consumer utility. It also allows us to use profit tax to raise tax revenue. Munk (1978) decreasing returns to scale, adding income from profit in the indirect utility function and profit tax revenue in the government revenue constraint. From the maximization of this problem the following modified Ramsey formula can be derived

$$\frac{\sum_{i=0}^{N} t_i S_{ik} + awS_{kw}}{X_k} = \left[\frac{\gamma^D - \lambda}{\lambda}\right]\left[1 - \frac{\frac{\partial \pi^c}{\partial p_k}(1-\tau)}{X_k}\right], \tag{30}$$

where π^c is the compensated profit function, i.e. the profit as a function of the consumer prices when keeping utility fixed.

We should now consider two cases. In the first, government revenue requirements are below the value of the total profit, allowing the government to raise all tax revenue through profit tax alone. Since the profit is perceived as exogenous income from the consumers point of view, the profit tax is obviously only working as a poll tax and must therefore be optimal. The left side of the formula then reduces to zero because of the equality between λ and γ^D. In absence of a profit tax one could tax profit indirectly by increasing all consumer prices in equal proportions or reducing all producer prices in the same proportions, which would mean taxing/subsidizing all goods. In other words, we cannot fix the tax rate a in this case, for instance, as implied by our earlier discussion about normalization.

In the second case, government revenue requirements are greater than the value of the profit. Here, we can use a 100 percent profit tax conjoined with indirect taxes. As the formula above shows, it would mean setting indirect taxes according to the standard Ramsey rule. Also in this case we could use other taxes in the absence of an profit tax. We would first calculate what the optimal tax structure and prices would be with an 100 percent profit tax, and then increase (decrease) all consumer prices (producer prices) by equal proportions, which would do the same job as the profit tax.

So far the demand side is the only thing that matters in the optimal tax formula, as was the case with the constant returns to scale situation. But if we assume that one good is untaxable and that no profit tax exists, equation (30) will apply for all goods except this one. Moreover, the relative reduction in compensated demand should then depend on how the taxes affect the profit. We see that it should decrease the compensated demand relatively more in optimum if the decrease in profit is relatively large. Since it is not possible to tax the sales of every good, one must try to tax the profit by using those tax rates which indirectly extracts profit in the best way, see Munk (1978).

2.1.2 When distribution comes into account

The models outlined in section 2.1.1 might not seem particulary interesting from a policy point of view. To add more realism to the models we must take distributional concerns into consideration (Diamond, 1975). We start by considering the constant returns to scale case with constant producer prices in a many-consumer economy. We assume no externalities and formulate the problem as

$$\max_{p,m} W(v_1(p_1,..,p_N,w_h,a,m,\mathbf{z}_1),..,v_H(p_1,..,p_N,w_h,a,m,\mathbf{z}_N)) \tag{31}$$

$$s.t. \sum X_i(p_i - q_i) + Hm = R.$$

We start by maximizing with respect to consumer prices. By proceeding as above (following Auerbach (1985)) we can reach a modified version of the equation (17), namely

$$\sum_{h=1}^{H} \beta_h x_i^h - \lambda \left[\sum_{h=1}^{H} \sum_{i=1}^{N} t_i \frac{\partial x_{hi}}{\partial p_k} + X_k + a \sum w_h \frac{\partial L_h}{\partial I} \right] = 0, \tag{32}$$

where $\beta_h = \frac{\partial W}{\partial v_h} \frac{\partial v_h}{\partial I}$, $w \frac{\partial L_h}{\partial I} = \frac{\partial Y_h}{\partial p_k}$ and x_{hi} is private demand for good i for household h. This needs interpreting so we define the social net marginal utility of income for consumer h as

$$\gamma_h^D = \beta_h + \lambda \left[\sum_{i=1}^{N} t_i \frac{\partial x_{hi}}{\partial I_h} + w_h a \frac{\partial L_h}{\partial I} \right]. \tag{33}$$

Then equation (32) can be expressed as

$$\frac{\sum_{h=1}^{H} \sum_{i=1}^{N} t_i S_{ik}^h + \sum_{h=1}^{H} a w_h S_{wk}^h}{X_k} = \frac{\gamma_k^* - \lambda}{\lambda}, \tag{34}$$

where

$$\gamma_k^* = \sum_{h=1}^{H} \left(\frac{x_k^h}{X_k} \right) \gamma_h^D. \tag{35}$$

Equation (34) shows that the relative reduction in compensated demand should be larger the lower the γ_k^*'s are. Note now that

$$\gamma_k^* = cov(\frac{x_k^h}{X_k}, \gamma_h^D) + \frac{1}{H} \sum_{h=1}^{H} \gamma_h^D. \tag{36}$$

This states that γ_k^* exceeds the (unweighted) mean of γ_h^D if and only if the covariance between the γ_h^D and demand for commodity k over individuals are positive. To expand on the interpretation, assume for a moment that $w_h = w$ and the existence of linear Engel curves with the same slope and separability between consumption and leisure exists such that the marginal propensity to pay indirect taxes, $\sum_{i=1}^N t_i \frac{\partial x_{hi}}{\partial I_k}$, could be written as $\sum_{i=1}^N t_i \frac{\partial x_i}{\partial I}$, and marginal propensity to pay direct tax, $w_h a \frac{\partial L_h}{\partial I}$, could be written as $wa\frac{\partial L}{\partial I}$, i.e., both are independent of h. Then the only term determining the level of γ_h^D is β_h, i.e. the social valuation of more income to individual h (see eq. (33)). Given a concave welfare function, if demand for the commodity is highest (measured as share of total market demand) for high income household, then we should reduce the compensated demand for this relatively more than when $\beta_h = \beta$. In other words, higher taxes on the commodities consumed more by the rich. Next, assume only that β_h is equal for all h, implying that the only term determining the level of γ_k^* is $\sum_{i=1}^N t_i \frac{\partial x_{hi}}{\partial I_k} + w_h a \frac{\partial L_h}{\partial I}$. In this case we would lower the demand for the commodity i more if it is consumed less by those with high propensity to pay taxes. Generally these distributional effects will interact. The former has to do with the social valuation of income to different individuals, i.e. the concern for equity, and the latter has to do with the concern of efficiency.

If the commodities are consumed in the same market proportions for every consumer or/and the γ_h^D are equal for all h, according to equation (36) zero covariance would obtain and the compensated demand should be reduced in the proportions suggested by the one-consumer Ramsey equation. The former requires the same level of consumption by consumers independent of income, which is impossible since it requires zero Engel elasticities. The latter requires either that β_h and $\lambda \left[\sum_{i=1}^N t_i \frac{\partial x_{hi}}{\partial I_h} + w_h a \frac{\partial L_h}{\partial I}\right]$ sum to the same for all h or that, more likely, the β_h and $\sum_{i=1}^N t_i \frac{\partial x_{hi}}{\partial I_h} + w_h a \frac{\partial L_h}{\partial I}$ are the same for all h, i.e., that valuation and propensity to pay taxes are equal for everyone. If valuation and propensity to pay taxes are equal for everyone we have an instance of the case studied above and the same results for uniformity and tax structure will apply here.

With linear Engel curves through the origin and separability between leisure and goods, each consumer will have the same budget shares for every income and, it follows, their market share will be the same for all commodities. Covariance will be equal for every commodity under these conditions, in which case we should lower the compensated demand in optimum equally for all commodities . The intuition behind this is that we cannot discriminate between the consumers and therefore should only care about efficiency effects of indirect taxes. What we end up with then is a uniform structure.

Further, as with our discussion above, notice that we are not given any explicit solutions for the individual tax rates.

When we optimize (31) with respect to the poll tax, Auerbach (1986) shows that (34) now becomes

$$\frac{\sum_{h=1}^H \sum_{i=1}^N t_i S_{ik}^h + \sum_{h=1}^H a w_h S_{wk}^h}{X_k} = \frac{cov(\frac{x_k^h}{x_k}, \gamma_h^D)}{\frac{1}{H}\sum_{h=1}^H \gamma_h^D}. \qquad (37)$$

This formula states that the reduction in demand should only depend on the extent to which the good is consumed relatively more by the consumers with low value on $\gamma_{h.}^{D}$. If the covariance is zero we should just employ a uniform poll tax. The covariance here could be zero, either because of equal market shares for each commodity or because of equal social valuation of income to individuals. Marginal propensity to pay taxes will only be zero in the case of a poll tax, so this term does not affect whether we should introduce indirect taxation. In practice, if we care about equity, then we should consider indirect taxation to supplement a poll tax. As before, we can not say anything about the tax structure without considering special cases. Nor does the formula tell us whether the indirect tax structure will become more or less progressive as the concern for equity grows.

When studying equation (37), we should note the special case where the covariance is equal for all commodities due to equal consumption patterns for all consumers. Since this requires linear Engel curves through the origin and weak separability, we know that equal reduction in compensated demand is implied by a uniform indirect tax system. In other words, we could implement indirect taxation although we cannot discriminate among consumers, simply because a uniform poll tax is more regressive compared with a uniform indirect tax system and less desired from a equity point of view.[13] Put differently, although we cannot discriminate with respect to tax payments between the consumers at same income level by using the indirect tax instrument, it is still worthwhile using it to change the total tax paid as share of income at different income levels. But, of course, we could only implement a proportional income tax instead, so the need for indirect taxes as an extra instrument does not really exist.

We now have sufficient conditions for a uniform indirect tax structure under the possibility of employing a uniform poll tax, namely linear Engel curves through the origin and the assumption of weak separability. Deaton and Stern (1986) have taken the case with linear Engel curves and separability as a point of departure for generalizing further. They assume consumers differ in preferences (and consumption patterns) partly due to differences in demographical characteristics (such as number of children) and partly due to idiosyncratic preference variation. Differences in preferences must here typically be represented by differences in the intercept of the Engel curves. They show that if we correlate only social valuation weights, i.e., β_h, with differences in preferences and market shares explained by the differences in the demographical characteristics, and the poll taxes could be conditioned on these demographic characteristics (in a linear way), a uniform indirect tax system is optimal. An example of specification is our equation (12), where

[13] Here we implicitly assume that the planner has some preferences for equity, i.e. have a concave welfare function. In the opposite case one would like to implement a uniform subsidy structure and finance this by increasing the poll tax beyond the tax revenue requirement. This makes the system even more regressive than in a pure poll tax system.

m is allowed to depend on number of children and adults. If then preferences differ by household composition and social weights are correlated with this composition, we only need to design an optimal m_h. We can then in some sense discriminate with respect to relative tax payments between consumers at the same income level simply by means of an efficient poll tax. In this sense a differentiated indirect tax structure would still be superfluous.

Deaton (1981) also examines indirect tax structure under the assumption of linear Engel curves. In the case of implicit separability between leisure and goods, he finds, the indirect tax structure is progressive if the government has preferences for equity. Some intuition for this could be gained by considering equation (37), where the covariance term has a relatively larger negative value if the commodity in question is consumed relatively more by those with low social value. In consequence, the compensated demand for luxury goods should be reduced relatively more. Implicit separability implies that the cross Slutsky elasticities between wage rate and other commodity prices are equal, so that decreasing the compensated demand for this good relatively more must mean that we tax this good more heavily, i.e. we implement a progressive indirect tax structure. Moreover, it also holds for the case without the possibility of poll tax when there exist preferences in favour of equity.

Decreasing returns to scale and profit We saw in the case without any distributional concerns that if the revenue requirement is less than the value of the profit, we could use only profit tax, and if it were not, we should implement a 100 percent profit tax and follow the Ramsey rule to raise the rest. In a many-consumer economy this is more complicated and could result in the coexistence of distortive indirect taxation and profit tax below 100 percent, because the effect of reducing profit income has different significance for each household (Dasgupta and Stiglitz, 1972).

If we assume an optimal profit tax of less than 100 percent, we should expect the supply side to affect the optimal tax rule throught supply elasticities (cf. equation (30)). As Iwamoto and Konishi (1991) shows, however, that the owner share, ω_f, of the firm also plays an important role. Firms that are mainly owned by wealthy households should in this way have their output taxed more. As a result supply side, demand side, owner shares and social welfare weights will all determine what the formula for indirect optimal tax structure will look like.

2.1.3 Summary

In Table 1-3 I try to sum up the discussion above. I focus on different separability assumptions between labor and commodities and assumptions about the Engel curves, thereby determining the configuration of the optimal indirect tax structure and which contributions to the literature that use these assumptions.

Table 1 sums up the main results for a one-consumer economy under the assumptions of no profit, constant producer prices and a proportional direct tax. Assumptions about separability are crucial. With no separability assumptions we cannot say anything about the indirect tax structure, indicated by a hyphen (-). Furthermore, weak separability results in a regressive structure, and implicit separability gives us a uniform structure. Note that with both weak and implicit separability we have homothetic preferences. Engel curves are then running through origo giving proportional Engel curves, making it impossible to have non-linear Engel curves.

In Table 2 we start considering the many-consumer economy. First assumptions are proportional direct tax, no profit, constant producer prices and homogeneous preferences. We also assume that the planner has preferences in favor of equity, i.e., he puts more weight on the low income households. As expected, in the general case we cannot in principle say anything about the indirect tax structure. The same applies with weak separability. With implicit separability and linear Engel curves, a progressive indirect tax structure could be expected to prevail since equal welfare weight would have resulted in a uniform structure. In the case with both weak and implicit separability a uniform structure would prevail since everyone has an equal budget share and this share is unaffected by changes in income.[14]

Table 3 considers linear direct taxation in the many-consumer economy under assumptions of no profit, constant producer prices, homogenous preferences, and planner has preferences for equity. In the general case nothing could be said.[15] When we add the assumption of weak separability and linear Engel curves, we get the result that a uniform tax structure prevail.[16] With linear Engel curves and implicit separability we get a progressive structure, while we in the case of both weak and implicit separability get uniformity.

While separability assumptions are the only things that play a role in table 1, the one-consumer economy, Engel curves come into play when we consider table 2 and 3, i.e., the many–consumer economy.

[14] In the case of heterogeneous preferences we could not be sure about this since budget shares could vary between individuals. If preferences are correlated with income (ability), non-uniform taxation could be optimal.

[15] Note also that we cannot say it is possible that the structure becomes more progressive with stronger preferences in favor of equity.

[16] Note that this also applies when differences in preferences are due to differences in characteristics that are observable and vary systematic with welfare weights.

Table 1. Implications of separability and Engel curve assumptions in a one-consumer economy with proportional direct tax, constant producer prices and no profit

Separability	Engel curves	Ind. tax str.	Literature
Non	Linear	-	Ramsey (1927), Corlett & Hague (1953) Baumol & Bradford (1970), Dixit (1970) Lerner (1970), Samuelson (1981)
	Non-Linear	-	
Weak	Linear	Regressive	Atkinson & Stiglitz (1972) Deaton (1981)
	Non-Linear	Regressive	
Implicit	Linear	Uniform	Deaton (1981) Besley & Jewitt (1995)
	Non-Linear	Uniform	
Weak & Implicit	Linear	Uniform	Sandmo (1974)
	Non-Linear	Not possible!	

Table 2. Implications of separability and Engel curve assumptions in a many-consumer economy with proportional direct tax, constant produces prices, no profit, homogenous preferences and planner having preferences in favor of equity

Separability	Engel curves	Ind. tax str.	Literature
No	Linear	-	Diamond & Mirrlees (1971) Diamond (1975), Mirrlees (1975)
	Non-Linear	-	
Weak	Linear	-	
	Non-Linear	-	
Implicit	Linear	Progressive	
	Non-Linear	-	
Weak & Implicit	Linear	Uniform	
	Non-Linear	Not possible!	

Table 3. Implications of separability and Engel curve assumptions in a many-consumer economy with linear direct tax, constant produces prices, no profit, homogenous preferences and planner having preferences in favor of equity

Separability	Engel curves	Ind. tax str.	Literature
Non	Linear	-	Diamond (1975)
	Non-Linear	-	
Weak	Linear	Uniform	Deaton (1981), Deaton (1979), Deaton & Stern (1986)
	Non-Linear	-	
Implicit	Linear	Progressive	Deaton (1981)
	Non-Linear	-	
Weak & Implicit	Linear	Uniform	
	Non-Linear	Not possible!	

2.2 Indirect taxation in the presence of non-linear income tax: the Mirrlees tradition

So far we have been considering a situation in which the direct tax function $T(y)$ is either proportional or linear. In the next step we remove this restriction. To do so we must include a compatibility constraint. Extending our scope also means that the direct tax scheme facing consumers can no longer be described by only one or two parameters. Throughout this section we assume constant returns to scale, implying no profit, and no externality generating good.

2.2.1 The benchmark model

For simplicity's sake we assume there exist only two individuals, different only by virtue of facing two different wage rates; see Stiglizt (1982). The government knows the distribution of wage rates but can not observe the individual wage rates and is therefore unable to implement the first best solution. It must therefore offer the consumers different bundles of disposable income and earnings to get them to reveal themselves by their action. In other words, we are facing a problem of asymmetric information where wage rate information is private and known only to the consumer. Producer prices are assumed to be fixed. Formally we can express it as

$$\max_{p,Y,M} W(V(p_1,..,p_N,w_1,M_1,Y_1),\ V(p_1,..,p_N,w_2,M_2,Y_2)) \tag{38}$$

$$s.t$$

$$\sum_i X_i(p_i - q_i) + \sum_h T(Y_h) = R$$

$$V(p_1, .., p_N, w_2, M_2, Y_2) \geqslant V(p_1, .., p_N, w_2, M_1, Y_1),$$

where $w^2 > w^1$.

The first order conditions for this problem suggest a *marginal effective tax rate* equal to zero for the consumer facing the higher wage, and a marginal effective tax rate greater than zero for the consumer facing a lower wage (Edwards et al., 1994). Without any tax mix, i.e. only income taxation, marginal effective tax rate is, of course, equal to marginal income tax. The intuition behind this has to do with the asymmetric information. As the literature states, we have to induce some distortion in some cases to fulfill the self selection constraint. In other word, the lack of information is costly to the planner. Several studies have explored the shape of this optimal income tax scheme (Diamond, 1998; Ebert, 1992; Kanbur and Tuomala, 1994; Mirrlees, 1971; Sadka, 1976; Seade, 1977, 1982; Tuomala, 1984, 1990). Since we are focusing on indirect taxation, we will not go pursue this issue further, turning instead our attention to the role of indirect taxation under this regime. Edwards et al. (1994) derive from the first order conditions of the problem above that the following must hold to yield Pareto efficient taxation[17]

$$\sum_{i=1}^{N} \sum_{h=1}^{2} t_i S_{ik}^h = \Psi(C_k^1 - C_k^{2^*}), \tag{39}$$

where Ψ is the Lagrange multiplier associated with the self selection constraint. C_k^1 is the compensated conditional demand of good k for consumer 1 when he takes the bundle of disposable income and earnings intended for him. C_k^{2*} is the compensated conditional demand of good k for consumer 2 when he mimics and takes the bundle intended for consumer 1. They are conditional in the sense of conditioning on earnings. This formula resembles the Ramsey formula and states that (in optimum) the change in aggregated compensated (conditional) demand following a small intensification in the indirect tax system should be proportional to the amount by which the demand from consumer 1 exceeds that of the mimicking consumer 2. In other words, indirect taxation is of no use if the consumption patterns for equal disposable income (and hence earnings) are identical. In this context the two consumers have exactly the same preferences, so the only difference that can make up for different consumption patterns must be the difference in consumption of leisure due to the different wage rate. Given equation (39) we must have $\partial C_k / \partial w_h \neq 0$ if indirect taxation is needed.

[17] More precisely they maximize utility of household 1 keeping household 2's utility fixed. This is the standard approach im the literature. Nevertheless, since Pareto efficient taxation is a necessary condition for welfare optimum this applies for problem (38) as well.

Christiansen (1984) uses conditional Marshallian demand functions within a marginal reform framework in much the same manner as Corlett and Hague (1953). He works with a continuum of individuals and assumes the pre-existence of an optimal non-linear income tax. He asks whether an (marginal) introduction of indirect taxes/subsidies would increase the welfare. His demand functions are conditional in the sense of conditioning on disposable income; they are written as $X(p, M, -L)$. If $\partial X_k/\partial L \neq 0$ then indirect taxation is preferable, he concludes. Moreover, $\partial X_k/\partial L > 0$ calls for taxation and $\partial X_k/\partial L < 0$ requires a subsidy. One might also show that $\partial X_k/\partial L = -(\partial C_k/\partial w)(w/L)$ and further that $\partial X_k/\partial L = 0$ and $\partial C_k/\partial w = 0$ if preferences are weakly separable.

That no indirect taxation is needed under the possibility of a non-linear income tax and weakly separable preferences between leisure and goods was first derived by Atkinson and Stiglitz (1976). It is known as the 'Atkinson-Stiglitz theorem'. To get some intuition of the result we should realize that it is related to the self selection constraint of the problem. By taxing more heavily those commodities that the mimicker would consume more of than the less able, we are also letting him pay more indirect taxes. We discriminate them with respect to tax payments and this relaxes the self selection constraint by making it less attractive to mimic, resulting in the possibility of reduced efficiency loss by decreasing the marginal effective tax rate for the less able (Stiglitz, 1982).

2.2.2 Some model extensions

Saez (2002) studies the possibility of heterogeneous preferences in the above context.[18] He uses the same approach as Christiansen (1984) and considers marginal welfare improving indirect tax reforms in the presence of an already existing optimal non-linear income tax. As a point of departure for exploring whether or not indirect taxation is desirable he derives the following formula

$$\begin{aligned} \frac{dW}{dt_k} &= -\frac{1}{\lambda}\sum_h \beta_h \left[x_k^h - \frac{X_k(Y^h)}{H}\right] + \\ & \sum_h T\,'(Y^h)\left[\frac{dY_T^h}{dT(Y^h)} \cdot \frac{dT(Y^h)}{dt_k} - \frac{dY_{t_k}^h}{dt_k}\right], \end{aligned} \tag{40}$$

where derivatives are denoted by subscript such that for instance $Y_{t_k}^h = \frac{dY^h}{dt_k}$. We can improve welfare if the first or second term in the expression is non-zero.

The first term is entirely a welfare effect. It could be zero for two reasons. First, if consumers at a given income level have equal consumer patterns,

[18] This differences in preferences can be interpreted as differences in household composition, **z.**

then the individual demand for good k will be equal to the average consumption of that good and the expression inside the brackets will be zero. Secondly, if the welfare weights are non correlated with the consumption pattern, the term will also turn zero. This welfare effect derives basically from the possibility of discriminating between consumers at the same income level and redistribute according to the social weights.[19]

The second term is a behavior effect. Saez shows this could be non-zero for the following reasons. First, it can be non zero because of dependence between consumption pattern and income and substitution effects. If labor supply responses change in a systematic way for different consumption patterns, indirect taxes will have a role to play. For instance, if the consumers with high preferences for a certain good also have a relative low labor supply response, this good should typically face a lower indirect tax rate. Second, it could be non-zero because of the fact that consumers at a given income level might have different consumption patterns reflecting differences in preferences due to differences in ability, i.e. wage rate, or simply because of non separability between leisure and commodity demand. The latter effect was discussed under the basic model. It simply helps us screening the consumers and thereby relaxing the self selection constraint. What is added here through the possibility of heterogeneous preferences allows consumption patterns to differ between the less able and the mimicker despite the presence of a weak separability assumption. One could therefore let differentiated indirect taxation relax the self selection constraint, without relying on the existence of weak separability.

In the benchmark model all the producer prices and the wage rate faced by the consumers where all assumed to be fixed. Naito (1999) relaxes this assumption when he purposefully introduces the supply side into the analysis. He considers an economy consisting of two consumers (one skilled and one unskilled) and two producers. Each producer uses two input factors in the production, namely skilled and unskilled labor. Each of them produces a commodity where one is skilled labor intensive in production and the other being unskilled labor intensive. Crucial to his analysis is the assumption of non-constant marginal costs since this makes the factor prices change when introducing taxes. Naito shows it is desirable to use differential taxes even with weak separability between leisure and commodities. The argument runs briefly as follows. Taxing the skilled labor intensive commodity reduces demand for and production of the item in question, freeing up resources for the rest of the economy. The structure of the production changes in so far as a larger part of the production now goes on in the unskilled labor intensive sector. As a result, the gap between the unskilled and the skilled wage rate narrows. This is essential to increasing welfare since it relaxes the self

[19] A concrete example: Higher welfare weight on households with children is making it attractive to lower taxes on childrens food.

selection constraint, i.e., makes it less attractive for the skilled individual to mimic. Moreover, by distorting the unskilled less we can increase the efficiency and welfare.

2.2.3 Summary

Table 4 sums up the results under assumption of non-linear income taxation. From the table we observe that under heterogenous preferences it will be desirable with a differensiated tax structure. With homogenous preferences, the crucial determinant is whether or not we have separability. With weak separability we get a uniform indirect tax structure.

Table 4. Implications of separability and Engel curve assumptions in a many-consumer economy with non-linear direct tax, constant producer prices, no profitt, and differing social marginal valuation of income

Separability	Preferences	Engel curves	Ind. tax str.	Literature
Non	Heterogeneous	Linear	Diff.	Saez (2002)
		Non-Linear	Diff.	
	Homogenous	Linear	Uniform	Mirrlees (1976)
		Non-Linear	Uniform	
Weak	Heterogeneous	Linear	Diff.	Saez (2002)
		Non-Linear	Diff.	
	Homogenous	Linear	Uniform	Atkinson & Stiglizt (1976), Mirrlees (1976), Christiansen (1984)
		Non-Linear	Uniform	
Implicit	Heterogeneous	Linear	Diff.	
		Non-Linear	Diff.	
	Homogenous	Linear	Diff.	
		Non-Linear	Diff.	
Weak & Implicitt	Heterogeneous	Linear	Diff.	
		Non-Linear	Not possible!	
	Homogenous	Linear	Uniform	
		Non-Linear	Not possible!	

2.3 Externalities and merit goods

The economies studied so far were absent of externalities or merit goods. We shall briefly in this section consider the effect of including these effects on the results for the tax structure.

2.3.1 Externalities

Sandmo (1975) was the first paper to consider optimal taxation with the possibility of an externality in the economy. He did this within the Ramsey framework with many identical individuals and derived for the representative household what he called the additivity property. Maxmizing (6) with respect to prices we get

$$-\alpha X_k + H\frac{\partial U}{\partial X_j}\frac{\partial X_j}{\partial p_k} + \lambda\left[X_k + \sum t_i \frac{\partial X_i}{\partial p_k} + a\frac{\partial Y}{\partial p_k}\right] = 0, \tag{41}$$

or alternatively we can write; see Auerbach (1986, pp.113)

$$-\alpha X_k + \lambda\left[X_k + \sum t_i^* \frac{\partial X_i}{\partial p_k} + a\frac{\partial Y}{\partial p_k}\right] = 0, \tag{42}$$

where

$$\begin{aligned} t_k &= t_k^* && \text{for all } k \neq j \\ t_k &= t_k^* - H\frac{\partial U}{\partial X_j}/\lambda && \text{for } k = j. \end{aligned} \tag{43}$$

The equation (42) is the standard optimum tax condition now applying to the taxes $(t_1^*, ..., t_N^*)$ instead of $(t_1, ..., t_N)$. The result could also be stated as follows: The externality effect only impact the optimal tax formula for the externality generating commodity, and it does this in an additive way. And, as noted by Sandmo (1975, p.92)

> ..the fact that a commodity involves a negative externality is not in itself an argument for taxing other commodities which are complementary with it, nor for subsidizing substitutes.

In this sense the optimal tax rates (t_i) and the externality correcting (Pigovian) tax rate $(t_i - t_i^*)$ are independent. We can imaging having chosen the optimal tax rates (t_i) first before correcting for the externalities with an additional correcting tax.

It is worth comparing this with the first best problem. A welfare optimum would in this case be characterized by just a tax on the externality generating good and the tax would be set equal to - $H\frac{\partial U}{\partial X_j}/\lambda$, i.e. the total marginal external cost. In this sense the result from the first best world carries over to the second best world.[20]

If we move to an economy where distributional concerns matters and where a lump sum tax is not possible, we would get a formula close to the one above. The additivity property still holds, but distributional concerns could impact the Pigovian corrective taxes since our social valution of a marginal decrease in utility for households can differ. For instance, if mainly poor households suffer from a negative externality, and the planner has preferences in favor of equity, then the corrective tax will tend to be large. In this way, distributional and efficiency concerns must be considered before introducing corrective taxes. This in contrast to the first best world where we can only set the tax equal to $-H\frac{\partial U}{\partial X_j}/\lambda$, and redistribute resources through a lump sum tax.

One should note the following. The independence between the optimal taxes and the corrective taxes is, as commented by Auerbach (1986), only present at an analytical level. The actual externality correcting tax rate depends on the optimal tax rates through the actual equilibrium and vice verca, making it hard to say anything about the tax structure in a problem where the externality is neglected compared to a problem were it is included. More precisely, it is difficult to suggest increasing the total tax rate on commodity x_j when the externality effect is included in our problem. Actually, in principle, we cannot know what the total tax rate will look like because by including a corrective tax we change the optimal taxes and, in extension, the total tax rate.[21]

The inclusion of an externality has also been carried out within the framework of non-linear direct taxation (Pirttilä and Tuomala, 1997). To understand the main mechanism it might be fruitful to start with the special case with weak separability between leisure and commodities. In this case we have no need for indirect taxation when we don't include externality effects. Let us first assume an externality effect is introduced and that the (dis)utility of this is additive separable from the other utility. We could specify the direct utility function as (Pirttilä and Tuomala, 1997, p.389)

$$U = a(u(x_{1,...}x_{N,}), L) + b(X_j). \tag{44}$$

[20] Of course, the marginal external costs in first best will not be the same as the marginal external costs in second best, because income and prices will differ in the both situations. But, the term as it is contained in the optimal tax formula is similar with the characterization of first best optimum. Bovenberg and de Mooij (1994) demonstrate that with already existing distortionary taxes the optimal corrective tax lies below the first best Pigovian tax rate.

[21] Much of the same results will apply in the case of a linear direct tax. See Pirtillä and Schön (1999) for a study on this. In the case of linear Engel curves and separability between leisure and commodity demand (Deaton, 1979; Deaton and Stern, 1986), we could completely let the direct tax take care of redistribution and use indirect taxes to fully internalize externalities.

In this case, as in the first best case, we can separate the distribution from efficiency effects and let the direct tax solve the redistribution problem. We should just set the indirect tax rate equal to the sum of marginal social damage as in the first best situation. The intuition behind this result has to do with the lack of impact on the self selection constraint. Recall that the only difference between the type 1 person and the mimicking type 2 person is that the latter enjoys more leisure. An effect on the self selection constraint must have been considered if we had complementarity between leisure and the externality. In other words, the impact on the self selection constraint, which in turn influences the ease of redistribution, must be considered when trying to internalize externality effects by indirect taxation. In this way the Atkinson-Stiglitz theorem is no longer valid since indirect taxation can prevail under the assumption of separability between leisure and commodity demand as well.

2.3.2 Merit goods

After 'merit goods' was introduced by Musgrave (1959), it has prompted several interpretations, as discussed by Besley (1988). Besley recognizes an approach which proceeds by exploiting the distinction between ex ante and ex post efficiency when examining an economy under uncertainty. Using it in the context of merit goods was suggested by Sandmo (1983), who investigates a case in which consumers misperceive the probabilities and end up with an ex post inefficient equilibrium. The reason is market failure, i.e., cases in which we have inefficient allocation of resources initially. A remedy in this context could be the provision of necessary information such that ex post efficiency prevails, i.e., it motivates governmental interventions based on market failure arguments.

The more common interpretation used by Besley (1988), is related to neglecting the notion of consumer sovereignty when considering the households valuation of consumption. To some extent this implies making households take decisions they would otherwise not have made. Typical examples of goods associated with such arguments are alcohol, tobacco (demerit goods) and education (merit goods). Moreover, we can assume that the planner values a given bundle of consumption and labor supply for household h according to the utility function $u_h^p(x_1, ..., x_N, L)$. In this way the problem for the planner is to

$$\max_{\mathbf{p}} u_h^p(x_1(p,w), ..., x_N(p,w), L(p,w)) \tag{45}$$

$$s.t. \sum_i t_i X_i = R,$$

where the demand function is derived from the households' maximization of their utility function, namely $u(.)$. Note that in this context the reason for intervention is not market failure (ex. imperfect information) but rather the failure for some reason of consumers to take appropriate choices from the planner's point of view.

One could imagine consumers supporting such a policy although they are perfectly free to choose whatever they desire. As Besley (1988) comments, it is possible to make a distinction between what consumers desire and what they value. The standard utility framework does not make this distinction. The consumers might know what really is valuable for them, but in lack of discipline they rather follow their desires. Governmental interventions can help them to choose what they really value.

Besley (1988) uses a scaling approach to model merit goods effects and he considers the effects in the light of what is relevant: individual consumers consumption patterns. In the first best solution this will result in a differentiated tax structure in the sense that tax rates differ from individual to individual. When we restrict tax rates by making them the same for all individuals we step into the second best world and merit goods effects along with distributional effect must be considered (as in the externality effect case above).

2.4 Optimal taxes and cross-border shopping

Cross-border shopping can become an issue when there is a price differential between countries. As Christiansen (2003) notes, price gaps could be due to tax differences, but also to other price determining factors such as differences in production costs, exchange rates and competitiveness of markets. When consumers are faced with such price differences they may find it beneficial to engage in cross-border shopping as long as the transportation costs are moderate. Cross-border shopping will impact the optimal tax rates. Consumers view both taxes and transportation costs as a resource costs. But taxes constitute only transfer of resources among different sectors in the economy, while transportation costs are real costs to society. Cross-border shopping is therefore costly to society, and should be taken into account when setting the optimal tax rates.

Within the context of welfare optimal tax design the number of cross-border shopping papers is limited.[22] One example is Christiansen (1994) which explore the optimum commodity tax on a good in a competitive and monopoly market. In a competitive marked the inverse elasticity rule is modified in the sense it is not total demand but rather domestic demand that matter. And as long as the domestic demand becomes less elastic when foreign prices increases, the optimal domestic tax rate/price is an increasing function of the foreign price (Christiansen, 1994, p. 333). Christiansen also consider two other cases, foreign and domestic monopoly respectively, and

[22] No one within the Mirrlees tradition of non-linear income tax has studied this.

derives modifications of the inverse elasticity rule within this context as well (Christiansen, 1994, p. 334-340).

Most of the literature studying cross-border shopping find themselves outside the framework of welfare optimal taxation. Instead they assume that the planners are tax revenue maximizing and that they then participate in a tax competition game. Two examples of such papers are Kanbur and Keen (1993) and Nielsen (2001).

2.5 Production efficiency

In our discussion so far we have taken for granted that we operate on the boundary of the production possibility set. In other words, we have implicitly assumed that the marginal rates of substitution of different factors in different firms are equal. This will be the case if the relative factor prices faced by the producers are equal. When knowing that production inefficiency means that we can get more output of at least one good without using more input, it seems very intuitive to assume that production efficiency will occur in an optimal designed regime consisting of linear distortive taxes. On the other hand, some knowledge about the literature on second best might make us suspect something else since the message here is that distortion one place might lead to the need for distortions elsewhere (Lipsey and Lancaster, 1956).

Diamond and Mirrles (1971) show that Lipsey and Lancasters message does not apply in the case where every commodity and input factor are taxable and where we have zero profit or, alternatively, that profit are taxed at 100 percent.[23] The logic goes as follows. If there exist no restrictions we can use the price structure to make the consumers demand a given bundles of goods on the production frontier.[24] In fact every technical feasible demand could be implemented by choosing an appropriate price structure facing the consumers. Obviously we would then not choose a point in the interior of the production set since this always would mean that we get less of at least one good. From this we can conclude that at the optimum in this case we must have production efficiency.

It turns out that for production efficiency not to be optimal, we must impose restrictions concerning the possibility of taxation, by assuming for instance that the commodities or/and input factors are not all taxable (Stiglitz and Dasgupta, 1971). Now not all technical feasible demands could be implemented since we cannot freely choose the consumer price vector, limiting one's freedom to implement every point on the production frontier. Following this logic we realize that a point in the interior of the production set

[23]Either by using a profit tax or, alternatively, using indirect taxation as explained above.

[24]Even though there are no restrictions on taxation, the same problem may occure due from lack of information, large administrative costs of identifying variables etc.

could be preferable.

In the constant returns to scale case we could, for instance, let one commodity be untaxable together with our untaxed numeraire commodity. This would give us one untaxed and one untaxable good. The point of abandoning production efficiency now has to do with the need to remedy this situation by taxing the untaxable good indirectly by taxing the input factors differently in different firms. Imagine we have an untaxable good produced by some firms. We might like to impose tax on input factors in these firms to serve as a substitute for the missing commodity tax, which should help obtain consumer prices (and demand) more in accordance with the case without restrictions. A trade-off between the desire for overall production efficiency and the desire to imitate the missing commodity tax will prevail (see Munk, 1980). If the input tax was able to serve as a good substitute in the untaxable commodity producing firms, the latter effect will predominate. Factors determining this are the elasticity of substitution and the factor share in the cost function of these firms. A low factor share combined with low elasticity of substitution will typically call for a high relative factor producer price here for the factor in question. Intuitively then we could change the consumer price indirectly as we see fit without inducing to much production inefficiency.

In the decreasing returns to scale case the same logic applies only now the need for taxing profit is the crucial point. As we recall, in the unrestricted case we could impose a uniform structure on all commodities and input factors and tax profit in a first best way. If one commodity proves untaxable, this is not possible any more. Profit could be taxed, however, by taxing the input factor which would increase the price of the final good. High final good taxes prices will narrow the real value of the profit, and the output will be reduced, which again will reduces the profit. Ceteribus paribus, higher output reductions will typically call for a higher relative factor price. As above the elasticity of substitution and the factor share in the cost function will also help determine how well the input tax can serve as a substitute for the missing commodity tax. Recall that under section 2.1 we also discussed the case with restriction under decreasing returns to scale and an implicit assumption of production efficiency. We saw there that we should impose relatively more tax on commodities where a producer price reduction resulted in a large decrease in profit. Now, by introducing the possibility of production inefficiency, we get more tools and are allowed to affect the consumer price and the untaxable good as well.

Next, what about the non-linear income case? Is production efficiency desirable? Answering them, Naito (1999) recourse to the model framework described in subsection 2.2.1 above to investigate the need for production efficiency in public production. He shows that the desire of overall production efficiency (in private and public sector) does not hold when considering such a model. Not surprisingly this has to do with the self selection constraint and the attractivity of mimicking. The reasoning is briefly the following. Let the government employ more unskilled labor than a production efficiency point of view would prescribe. It will create a shortage of unskilled labor and raise wage rates relative to the skilled worker rate, relaxing the self selection constraint and improve welfare. The same result and logic will apply in a model without public production but where certain input factors can be observed by the government and taxed by differentiated rates according to whether a firm is skilled labor intensive or not. The main point is nevertheless to affect the relative wage through changing factor prices faced by the producers.

2.6 Optimal marginal reforms

We have so far been occupied with finding globally optimal indirect taxes. Another approach could be to investigate marginal tax reforms, i.e. asking which small changes in tax rates would increase welfare in an economy with a given tax structure. A well-known paper in this regard is Ahmad and Stern (1984a)[25]. One could investigate as a point of departure the first order condition given for the optimal tax problem above. At the global optimum the following must always hold

$$\frac{\frac{\partial W}{\partial t_i}}{\frac{\partial R}{\partial t_i}} = \lambda \text{ for all } i, \tag{46}$$

i.e., that the marginal social cost of public funds should be equal for every commodity. Away from optimum this does not hold and we get

$$\frac{\frac{\partial W}{\partial t_i}}{\frac{\partial R}{\partial t_i}} = \lambda_i,$$

i.e., that the marginal cost of public funds depends on which source we chose to raise the revenue from. As long as $\lambda_i \neq \lambda_j$ for all $i \neq j$, there will be some welfare improving marginal tax reform.

Furthermore the marginal costs of public funds for commodity k can be written as

$$\frac{\sum_h \beta^h x_k^h}{X_k + \sum_h \sum_i t_i \frac{\partial X_i^h}{\partial p_k} + a \sum_h \frac{\partial Y_h}{\partial p_k}} = \lambda_k. \tag{47}$$

From this it follows, that λ_k could be calculated if we have the information on expenditure patterns, aggregate demand derivatives and tax rates in the initial situation as opposed to globally optimal tax rates that require far

[25]Other important works on this topic are Dixit (1979), Guesnerie (1977, 1995) and Hatta (1986). Besides these, a special case of this approach was considered as early as 1953 by Corlett and Hague.

more information to be calculated. This issue will be discussed in next section.

On the other hand, the marginal reform approach is rather limited in scope compared to the globally optimal tax approach. Firstly, we obviously cannot say anything about the size of the tax changes, only the directions. Secondly, there will in general be many marginal welfare improving reforms. Choice of direction is left open within the context of a marginal reform approach.

3 Applying the theory

As the previous section suggests the number of purely theoretical contributions within the field of optimal indirect taxation is very high. What then about empirical contributions? What attempts have been made to apply theory to data? What kind of models and methods have been used and what do the result suggest about the tax structure? This section is devoted to discussing these questions. But before probing the literature, I want to start by clarifying the kind of information we need to compute the optimal tax rates, and the kind of difficulties we might run into.

3.1 Informational requirement for computing optimal taxes

Let us start by assuming that producer prices are constant. The first order conditions and the budget constraint will make our need for information about the individuals' demand and their derivatives immediately apparent. We will also need information on individuals' supply functions, such as labor supply. A demand/supply system for the consumers must be specified and parameters determined from data by some methodology. Without imposing further restrictions such as commodity/leisure separability, we can now see how wage rate and leisure consumption influence demand for commodities through substitution and complementarity and not only income effects. After reading Browning and Meghir (1991) we should not be very optimistic about finding such studies. They write that "Almost all empirical investigations of demand and consumption assume that preferences over goods are separable from labor supply". We already know from the previous discussion that such an assumption will determine much of the result a priori, cf. Deaton (1981). Knowing that such separability is rejected in several empirical demand analysis (Blundell and Walker, 1982; Blundell and Ray, 1984), it does not seem like a very appealing restriction.

Regarding the model with optimal non-linear income taxation it should be obvious that we cannot rely on the assumption of separability between labor supply and commodities, since this determines the optimum to be a uniform system.

In line with this discussion, one could argue that the functional form should be flexible enough to allow for non-linear Engel curves. But on the other hand we should be aware of the following. Undertaking an optimal design analysis requires the demand and supply function to be consistent with consumer theory globally, not only locally as the optimal price structure could be far from the point at which the function is consistent with theory. So-called flexible functions do not automatically exhibit these properties in a global sense. The property of a flexible function form is its ability to take on any set of price and income elasticities at a particular data point, i.e., unrestricted by a priori assumptions. This seems very desirable, but it comes at a cost as Caves and Christensen (1980, p. 423) make clear :

> A flexible form can achieve arbitrary price and income elasticities at any particular data point. However, once a set of parameter values is chosen, either a priori or by statistical estimation, the pattern of price and income elasticities is determined for all possible data points. It is possible that the pattern of elasticities is plausible only for a limited range of data points, that is, that outside of a limited range of data the estimated indirect utility function is not monotonic and strictly quasi convex. In fact the range may not even include points in the sample under consideration.

An example of a popular and widely used flexible functional form is the almost ideal demand system (AIDS) introduced by Deaton and Muellbauer (1980). This function is an approximation to any arbitrary utility function and exhibits many desirable properties locally, including those imposed by theory. However, since it is an approximation it could only show consistence with theory locally, and we have no guarantee of the same applying globally.

Secondly, the social welfare function must be further specified in the case of many-consumer economy. Simplifications can be done by assuming an utilitarian welfare function and letting aggregation be possible. That leaves us with a representative consumer case, in which aggregate demand and derivatives are enough. The issue about global properties will remain, however.

Thirdly, we need non-trivial information on current effective tax rates. Ahmad and Stern (1984b) show how this can be done, they find effective tax rate on final goods.[26]

When all the information needed is at hand, we can calculate the optimal taxes. This is no trivial task since taxes, prices, quantities and elasticities are interdependent in a non-linear way. Some kind of numerical method must be employed to yield a solution.

[26]The taxes are endogenous in the optimal tax problem, but we need information of these to derive pre-tax prices (producer prices).

If we relax the assumption of constant producer prices the computation of optimal taxes will involve obtaining information about the producer sector behavior. In other words, at the end of the line we need some sort of general equilibrium model to have any hope of finding the optimal taxes within such an context. The analysis of the optimal taxes then becomes very complex, especially if we also are concerned with distributional effects in a many-consumer economy.

Let us turn to the different empirical contributions.

3.2 Empirical contributions

To my knowledge no attempts have been made at applying the theory of non-linear income tax along with commodity tax. We will therefore only review studies under the Ramsey model, i.e., linear or proportional taxation. The reviewed studies only consider the demand side, namely by assuming constant producer prices. Let us start with a series of contributions all of which is considering a one-consumer economy, i.e., they are occupied with purely efficiency aspects.

3.2.1 One-consumer economy - efficiency

Atkinson and Stiglitz's (1972) paper is the first, to my knowledge, to compute optimal tax rates from empirical data. In computing optimal taxes for five commodity groups they consider two demand systems: The linear expenditure system (LES) based on estimates by Stone (1954) and the direct addilog demand system based on estimates by Houthakker (1960). In both cases separability between leisure and other goods prevails and they assume a completely elastic labor supply. According to theory they should get a system which is regressive, and so they do. Their results conclude with high taxes on necessities and low taxes on luxuries, i.e., a clearly non-uniform system.[27] As we know an inelastic labor supply within this framework would have resulted in a uniform structure, it seems logical to suggest that a lower labor supply elasticity could change this picture. Fukushima and Hatta (1989), using the same data set and the same model, find that reducing the (compensated) labor supply elasticities works in favor of a uniform system. With what they consider as more reasonable values they find the structure to be fairly uniform. [28] Harris and McKinnon (1979) also calculated the optimal tax rates for five groups using a Stone-Geary function with different goods inside a CES function consisting of leisure and the goods. The structure, they conclude, varies with the assumption of substitution between leisure and other goods, namely the compensated labor supply elasticity. Fukushima (1991) uses the same data but with a lower

[27] For Sweden the optimal tax rates ranged from 11.1% on durables to 42% on food.

[28] Tax rates in Sweden ranging from 24.9% on food to 22.7% on durables.

labor supply elasticity, which, he finds, gives a result somewhat closer to a uniform system.

These models are very simplistic in considering only five groups and assuming separability between leisure and commodities. Asano and Fukushima (2006) estimate the joint decision of leisure and commodity demand without imposing any separability restriction. They use Deaton's AIDS and compute optimal tax rates for ten commodity groups in Japan. Their conclusion is that the optimal structure is reasonable close to a uniform structure which suggests that the welfare losses associated with a uniform structure are small. This is indeed confirmed when they calculate the welfare in the two situations for comparison. Although Asano and Fukushima's model provides important improvements, including increasing the number of goods, not imposing separability assumption, and non-linear Engel curves, what weakens their results is uncertainty about the extent to which the model complies with properties derived from consumer theory. The AIDS do not exhibit these properties for all prices and income. Despite performing a check on theory consistence for the estimated elasticities at the sample mean, they do not attempt to clarify, or even mention whether the same conditions will be fulfilled at optimum. This weakens their argument, but, as will be shown below, the same applies to other studies. Let us turn to studies within the context of a many-consumer economy.

3.2.2 Many-consumer economy - distributional concerns

As far as I am aware, the first example of calculation of optimal taxes within a many-consumer framework is Deaton's (1977). Deaton's model relies heavily on simplifying assumptions. He realizes the large informational requirements from the many-consumer Ramsey rule and finds an alternative path. By employing what he calls *strategic aggregation,* he ends up only having to consider two consumer's behavior, namely, in his vocabulary, the *social representative consumer* and the *average consumer.* He calculates optimal taxes for eight goods. His result also relies on inelastic labor supply and linear Engel curves. His specification of the welfare function is based on Atkinson (1970), and is similar to the welfare functions used in the studies reviewed below. Since theory tells us this would yield a uniform structure in the case where only efficiency matters, his results are not surprising. He mainly finds that when the concern for equity increases, the structure becomes more differentiated and luxuries are taxed more heavily than necessities.

Heady and Mitra (1980) use a Stone-Geary utility function, implying both separability and linear Engel curves for nine goods including leisure. Basically they find that the structure is highly non uniform no matter what the assumption of equity is.

Sensitivity of optimal tax rates to different demand systems is considered by Ray (1986). He calculates optimal tax rates for nine goods conditional on the prices, income and elasticities observed at a particular time. In other words, his optimal tax rates are not optimal in a strict sense but merely answer the question of what the tax rates would have been had the current situation constituted an optimum. He compares the linear expenditure system (LES) with the restricted non-linear preference system (RNLPS), which is a specialization of the non-linear preference system (NLPS) introduced by Blundell and Ray (1984). The RNLPS allows for non-linear Engel curves. He finds that the two demand systems agree at low level of concern for equity, but diverge when the concern for equity increases. At low levels of equity concerns they approach a uniform system.[29]

Ray and Murty (1989) develop an algorithm based on the marginal tax reform approach of Ahmad and Stern (1984) and calculate optimal tax rates. They use the general functional form of Blundell and Ray (1984), the non-linear preference system (NLPS), and investigate the sensitivity of relaxing the assumption of separability between leisure and other goods. Their results indicate that the tax rates for the nine goods considered are highly sensitive to deviations from the separability assumption. All their results show a non-uniform tax structure. Ray and Blacklow (2002) extend this and incorporate demographical effects when they use RNLPS and its specialization LES to study optimal taxes in Australia for nine goods. The optimal tax rates, they conclude, move away from uniformity when demographics are introduced, and the effect is more significant when considering the RNLPS than LES. In line with Ray and Murty's (1989) findings, their result also indicates that LES and RNLPS agree at low inequality aversion but disagree at higher inequality aversion. Further, the optimal tax rates would appear to be more sensitive to choice of functional form than to the inclusion of demographical effects.

Ray (1989) and Ebrahimi and Heady (1988) investigate the effect of demographical effects and child benefits. Both of them use data from a UK data base covering four consumer goods. Ray (1989) allows for non-linear Engel curves but does not include leisure, while Ebrahimi and Heady (1988) include leisure but use linear Engel curves. Ray (1989) finds support for non-uniformity, especially when inequality aversion is high. Ebrahimi and Heady (1988) conclude that that the separability assumption determines to a large extent the structure when demogrants (lump sum transfers conditioned on demographical characteristics) are not set optimally, in line with the theoretical contribution of Deaton and Stern (1986).

Revesz (1997) uses LES first with nine goods, then with 9 goods for the poor and 18 for the rich. In both cases leisure is separable from other goods

[29] Note that because of the non-linearity of Engel curves for the RNLPS, the marginal utility of income could differ between individuals because income levels differ. It would make distributional effects relevant and tend to give a non-uniform system, although the planner assigns equal weights to every individuals' utility.

and the possibility of employing transfers is present. In the first case he gets results in line with Deaton and Stern (1986), namely uniformity. When applying the second system he gets a non-uniform structure because of the non-linearity in the Engel curves.

Asano et al. (1994) use a AIDS with separability assumption between leisure and other goods. When lump sum transfers are not included they find that necessities should be taxed more heavily, but after including the possibility of an optimal lump sum they show that this property is reversed, i.e., a regressive commodity tax structure prevails and commodity taxes play no role as a redistributive device. Constraining the lump sum transfer (non-optimal) produces similar patterns to without lump sum transfers.

3.2.3 Summary

As with the theoretical results, we sum up the empirical literature and results in Tables 5, 6 and 7 below. In the table where we assume a one-consumer economy I have left out implicit separability. The category is uninteresting from an empirical point of view since we know from theory that it has to result in a uniform tax structure. For the same reason, 'weak & implicit separability' is not included in the many-consumer economy context. With regard to empirical studies using the assumption of inelastic labor supply in a many-consumer economy, I have put these together with weak separability, since under this assumption we must have separability.[30]

Table 5. Implications of separability and Engel curve assumptions in a one-consumer economy with proportional direct tax, constant produces prices and no profit

Separability	Engel curves	Ind. tax str.	Literature
Non	Linear	-	
	Non-Linear	Slightly non-uniform	Asano & Fukushima (2006)
Weak	Linear	Progressive 1)	Atkinson & Stiglizt (1972), Fukushima & Hatta (1989), Harris & Mckinnon (1979), Fukushima (1991)
	Non-Linear	-	

1) The degree of progressivity depending on labour supply elasticity.

[30] Computing optimal taxes taking labor supply and hence income as exogenous must rely on the assumption that only income counts for the demand patterns. Implicitly one then assume weak separability between leisure and goods.

Table 6. Implications of separability and Engel curve assumptions in a many-consumer economy with proportional direct tax, constant produces prices, no profit, homogenous preferences and planner having preferences in favor of e

Separability	Engel curves	Ind. tax str.	Literature
No	Linear	Non-uniform	Ray and Murty (1989)
	Non-Linear	Non-uniform	
Weak / inelast. lab.	Linear	Non-uniform 1)	Deaton (1977), Heady & Mitra (1980) Ray (1986), Ray & Blacklow (2002)
	Non-Linear	Non-uniform 1)	Ray (1986), Ray & Blacklow (2002), Ray & Murty (1989)
Implicit	Linear	-	
	Non-Linear	-	

1) When including leisure: Highly non-uniform even when equity does not matter. Not including leisure: More uniform when inequality aversion is low and progressive else.

Table 7. Implications of separability and Engel curve assumptions in a many-consumer economy with linear direct tax, constant producer prices, no profit and differing social marginal valuation of income

Separability	Engel curves	Ind. tax str.	Literature
Non	Linear	Non-uniform 1)	Ebrahimi & Heady (1988)
	Non-Linear	-	
Weak / inelast. lab.	Linear	Uniform	Revesz (1997)
	Non-Linear	Non-uniform 2)	Ray (1989), Revesz (1997), Asano et al (1994)
Implicit	Linear	-	
	Non-Linear	-	

1) Separability assumption important when demogrants are not set optimal.
2) More uniform when inequality aversion is low, assuming inelastic labor supply (cf. table 6).

3.2.4 Some critical remarks

After considering early empirical work it seems the demand systems used relied on very strict assumptions, such as separability and linear Engel curves. That does have the advantage of being perfectly theory consistent. More recent studies have avoided putting a priori assumptions on behavior, by using flexible forms. The results risk inconsistency with the theoretical foundation. Although it appears to be neglected in the literature, several examples can be given.

The study of Asano and Fukushima (2006) shows important improvements (such as increasing the number of goods and not imposing separability assumption together with non-linear Engel curves) but their results are weakened by the fact that nothing is done in order to check global characteristics at optimum. They discuss theory consistence for estimated elasticities at the sample mean, but make no attempt at clarifying or even discussing whether the same conditions will be fulfilled at optimum.

Asano et al. (2004) write that "the estimation results display consistency with demand theory, and elasticity estimates are close to economic sense", but seem to forget the next step to use this demand system and find globally optimal tax rates and price structure.

The same critic applies to other studies as Ray and Murty (1989) and Blacklow and Ray (2002). The flexible form demand system NLPS could not be said to be consistent with theory globally (Blundell and Ray, 1984, p.802). How their restricted versions (as RNLPS) perform with respect to this is not commented upon at all.

A second line of criticism has to do with the realism of the models and the use of the term 'optimal'. Even though the researchers do not incorporate externalities or/ and merit goods considerations, they still might emphasize that the actual tax rate on some goods are far from optimal. Without recognizing that this good (such as alcohol) have a high tax because of reasons left out of the model. If we want to find tax rates that is welfare optimal, we must recognize that the real world consists of indirect taxes that are based on specific considerations.

Thirdly they all adopt approaches which can be very demanding when it comes to information. Estimating a complete demand system when the number of goods increases, can be too demanding.[31] It seems that we then have to make some restrictive assumption and consider applying alternative methods to determine all the parameter values. Different types of calibration methods can perhaps be fruitful in this respect. An example of such an approach is given in Aasness and Nygård (2008).

3.3 Other empirical studies - optimal marginal reforms

As already noted, another approach to tax problems is to consider marginal reforms instead of globally optimal tax designs. From an empirical point of view, its advantage is the considerably lower information need. Where the globally optimal tax approach demands knowledge about individuals' complete demand functions, the marginal reform approach only needs information about individuals' consumption expenditures, aggregate demand derivatives and tax rates in the initial situation.[32] The marginal reform approach also seems to be more robust to choice of specification than the

[31] For instance in the case of 30 goods: we have to estimate 30x30=900 own-and cross price elasticities.

[32] Within the reform approach it is also possible to not specify a certain welfare function. One could search for Pareto improving reforms or, as in Yitzhak and Slemrod (1991), seek reforms satisfying a wider class of welfare functions.

globally optimal tax approach (Madden, 1995; Decoster and Schokkaert, 1990; Ray, 1986; Ebrahimi and Heady, 1988). All this makes marginal reform analysis attractive, although it remains somewhat limited in scope.

Ahmad and Stern (1984a) calculates marginal social costs associated with commodities in India when labor supply is taken as given. This allows them to say something about the direction of marginal reforms in indirect tax structure. Decoster and Schokkaert (1989) present a study of marginal reforms in indirect taxes for Belgium. Labor supply is fixed and marginal social costs are calculated for twelve commodity groups. Marginal social costs associated with alcohol and heating turn out to be high, independent of the degree of inequality aversion. This they realize has to do with merit goods arguments. They also compute the merit goods effect given that the tax system is optimal, and find that they would be considerable.[33] Aasness and Schroyen (2006) compute marginal social costs within the same framework as above where they also consider externality effects (greenhouse gases) and merit goods' effect.

All these studies neglect the effect on labor supply and direct taxes. One simple extension would be to include the effect on labor supply and direct tax payments as done in equation (47) and take the direct tax rate as given. We could also be interested in the marginal social costs associated with the direct tax rate. Madden (1995) includes leisure and direct tax and computes marginal costs for indirect taxes and the direct tax. Mainly, he concludes that the inclusion of labor supply does not alter the picture much, i.e., the ranking of commodities based on social marginal costs is rather robust. In particular the ranking does not seem very sensitive with regard to separability between leisure and commodity demand. Thus, his results contrast with the literature on optimal tax designs. As he also finds, the marginal social costs associated with the direct tax are at the lower range of estimated social costs, but this is reversed at very high levels of inequality aversion.

4 Conclusions

This paper has reviewed theoretical and empirical results within the context of welfare optimal indirect taxation. The first part presented theoretical contributions identifying the kind of tax rules and indirect tax structures that would prevail according to theory under different assumptions of the direct tax function. It turns out that for theory to tell us something about the properties of the indirect tax structure, we have to put rather restrictive assumptions on the preference structure.

But should the structure be differentiated or simply be uniform? By way

[33] In other words they solve the inverse optimum problem, see Christiansen and Jansen, 1978.

of an answer we can ask ourselves whether the indirect taxes add something to our toolbox, i.e., broaden the scope of tax instruments available. In the case of a proportional or linear income tax this can happen in two ways: The indirect tax instrument allows us to discriminate between income levels in the sense that tax payments measured as share of income changes or/and the indirect tax instrument could make it possible to discriminate between different consumers at the same income level in the sense that tax payments measured as share of income differ. The former would be possible when the budget shares change with changing incomes, and the latter when the budget shares differ between individuals at the same income level. This constitute a necessary condition for a differentiated indirect tax structure within the context of a proportional or linear direct tax.

In the case of a non-linear direct tax the indirect tax instrument can still add something, in the sense of making it possible to discriminate between consumers at the same income level. But since the direct tax is unrestricted, it will be so flexible that indirect taxation will not add anything in the sense of discriminating consumers' tax payments at different income levels. From this it follows that a necessary condition for differentiated indirect tax structure within the context of a non-linear direct tax is that consumers at the same income level must differ in budget shares.

When the necessary condition for differentiated tax structure is violated we obviously get a uniform structure. But a uniform structure could, of course, still prevail when the necessary conditions for differentiated structure are fulfilled (i.e., they are not sufficient conditions). In section 2.1.1 we saw how implicitly separable preferences between leisure and other goods constitute a sufficient and necessary condition for a uniform structure in a one-consumer economy with a proportional direct tax. In section 2.1.2 we studied the many-consumer economy and saw that if differences in budget shares at the same income level reflects differences in preferences, and this preferences are correlated with observable characteristics, we could employ a direct tax where we condition on the characteristics. The indirect tax structure will then be uniform.

We also discussed cases where indirect taxation is used not only for fiscal or redistributive reasons but to correct for externalities or merit goods. It gave us an important lesson: Whenever we can let the direct income tax completely take care of redistribution (i.e., the optimal indirect tax structure is uniform without the externality effect), indirect taxes could be used entirely to correct for such effects; namely we set indirect taxes as in first best with full internalization.

The second part of this review focused on the much smaller number of empirical contributions. The reason is probably the large amount of data needed to compute optimal taxes. Nevertheless, some studies do exist. They rely mostly on rather restrictive assumptions of preference structure, although some do use flexible functional forms. I have pointed to what

I believe is a dilemma in this respect. A researcher will be reluctant to make too many assumptions about preferences, because it could impact the empirical results a priori. But he needs to be careful about using functional forms that are not consistent with theory globally, since we are interested in finding globally optimal taxes.

5 Literature

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