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IMPLIKASJONER FOR ANALYSE AV YRKESDELTAKING NÅR DET TOTALE TILBUD AV ARBEID ER LATENT

BY/AV JOHN DAGSVIK

STATISTISK SENTRALBYRÅ
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PREFACE

The present report is a contribution to the theory of individual choice behaviour when the choices are made at different points of time and the alternatives are discrete.

With the analysis of labour supply in mind the theory is modified to cover the case where the set of available alternatives from which each individual chooses is not observed.

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Central Bureau of Statistics, Oslo, 28 April 1980

Petter Jakob Bjerve

FORORD

Dette notatet er et bidrag til teoriene for individuell valghandling når valgene skjer ved flere tidspunkter og tallet på alternativer er endelig. Med henblikk på analyse av tilbud av arbeid er teorien modifisert til å dekke situasjonen hvor settet av tilgjengelige alternativer, som hvert individ velger fra, ikke kan observeres.

Til dette arbeidet har forfatteren mottatt støtte fra Norges almenvitenskapelige forskningsråd. Det er delvis utført under studieopphold ved Department of Statistics, University of California, Berkeley.

Statistisk Sentralbyrå, Oslo, 28. april 1980

Petter Jakob Bjerve

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INNHOLD

1. INTRODUCTION

The traditional theory of consumption deals with choice behaviour of individuals facing a continuum of possibilities. Yet, in important applications, the set of commodities is discrete. Examples of this kind are choices conserning where to live, transportation mode, labour force participation, directions and level of education. If the choices are consistent, i.e. if identical choice settings yield identical outcomes the discrete case can be handled in essentially the same way as the continuous case. However, in most applications it is seen that individuals in observationally identical choice situations behave inconsistently, i.e. they do not make the same choice at every identical occasion. Since the alternative are discrete, small changes in the preferences may yield a complete change of the choice outcomes. It may therefore be important to modify the deterministic utility theory.

Probabilistic models for discrete choice to account for observed inconsistencies in choice behaviour have already a long tradition among psychologists. In the last decade there has been a rapidly increasing literature where economists attempt to incorporate these ideas and their implications in economic analysis.

The present paper describes one plausible extension of the present probabilistic choice theories to the dynamic case where individuals make discrete decisions at different times. The point of departure is the theories which have emerged from Luce's fundamental monograph on individual choice behaviour (1959) and in particular the contribution by Yellott (1977).

We consider a population of individuals characterized by a vector $x(t)$ of observed characteristics at time t. The individuals are consistently faced with a set of discrete alternatives called the state space,and they make a (new) choice when the ranking of the utilities have changed. The attractiveness of an alternative is measured by a utility function which depends upon time, the individual's and the particular alternative's observed characteristics. The utility function is observationally random due to unobserved characteristics of the individual and of the alternatives. In the dynamic case the utility function fluctuates randomly over time, i.e. it is a random utility process. Thus, if an individual is in the i-th state he will jump to another state j when the utility of the j-th alternative becomes larger than the utilities of the competing alternatives. The utility function at time t may depend upon previous choice history because past experience may alter future preferences. For instance, the occupied state may have an additional positive constant utility component attributable to the cost and psychological stress linked to a change of state. This type of dependence upon previously realized choices is here termed structural state dependence (cf. Heckman (1978b)).

While in a standard approach the population is assumed to be homogeneous (often implicily) we believe that the effect of unobserved individual characteristics may be important and the model should therefore be sufficiently general to account for population heterogeneity. Assumptions about population heterogeneity seem to have appeared for the first time in an analysis of occupational mobility by Blumen, Kogan and McCarthy (1955). They formulated a model assuming that an unknown fraction of the population were stayers and the rest were movers. Movers are individuals who change status in contrast to stayers who never move. They found that this model gave a better description of the data than the model based on the homogeneous population assumption. Their mover-stayer model has later been generalized to continuous time with different types of movers.

In the present approach the following explanations of population heterogeneity are considered:
1) The random term of the utility process shows different degree of temporal stability across observationally identical subgroups. As a consequence, the choice model for the representative individual may differ from the models followed by each individual in the group. This is the mover stayer analogue.
2) In applications such as the analysis of labour force participation, directions and levels of education, the set of available alternatives each individual faces is often not observed. For instance, an observed transition from "not employed" to "employed" may either be a result of a change in the preferences between the two alternatives or by an expansion of the choice set from containing only the alternative "not employed" to containing both alternatives (for an individual who wants employment).

To analyse cases where the choice sets are not directly observable we have introduced the random choice set concept. With special reference to the study of labour force participation, this device is operationalized by specifying probability distributions for changes in a particular individuals choice possibilities. The corresponding heterogeneity effect may be important if the choice sets vary across observationally homogeneous subgroups, but change slowly for each individual.

The parameters of the probability distributions connected to the random choice sets are estimated simultaneously with the other parameters of the model. This approach opens up for the possibility of estimating and predicting the conditional choice probabilities given the available alternatives. Notice that observations on the conditional behaviour given the choice set are not needed. In labour force analysis this means that the latent supply of labour can be predicted from observations on realized labour market transitions together with observations on the explanatory variables.

While "leisure" or "demand" indicators may explain some of the effects due to heterogeneity in the demand and supply functions across the population the present technique relies upon probabilistic assumptions made a priori the introduction of explanatory variables and offers a method for dealing with additional unobserved heterogeneity.

As noted by several authors (cf. Heckmann 1978b) population heterogeneity may induce dependence on past choice realizations at the aggregate level which differ from the corresponding dependence at the individual level (Spurious state dependence). For instance, the individual choice models may be independent of previous realizations and yet the model for the representative individual may depend on previous choice experience. An important problem is to estimate and test spurious versus (true) structural state dependence effects. However, the related inference problems are not discussed in the present paper.

Despite their obvious importance, dynamic probabilistic models incorporating exogenous information have obtained little attention in the literature. The most important contributions seem to be those of Coleman (1964), Heckman (1978a), McFarland (1970), Spilerman (1972a, b) and Ginsberg 1971, 192a, b). However Ginsberg is the only one using probabilistic choice theories to generate the functional forms of the parameters in his semi-Markov mobility model. His model is nevertheless only partly based on these theories and it is restricted to being time stationary and to homogeneous subgroups of observationally identical individuals.

This paper is organized as follows: In section two we describe in detail why the traditional consumer theory needs modification when the set of alternatives is discrete and in section three we describe certain models of individual choice behaviour.

In section four the essential parts of a dynamic model for individual choice behaviour is described and in section five the heterogeneity hypotensis are discussed. However, the formal development of the dynamic choice theory is given in a separate paper (Dagsvik (1980)).

In static choice theories attempts have been made to relax the choice axiom introduced by Luce to permit correlation between utilities for different alternatives. A possible extension to the dynamic case is sketched in section six. The final section deals with the application to sequential labour force participation analysis where the random choice set device is discussed in detail.

In a following paper we intend to deal with estimation of the models set forward in section six.

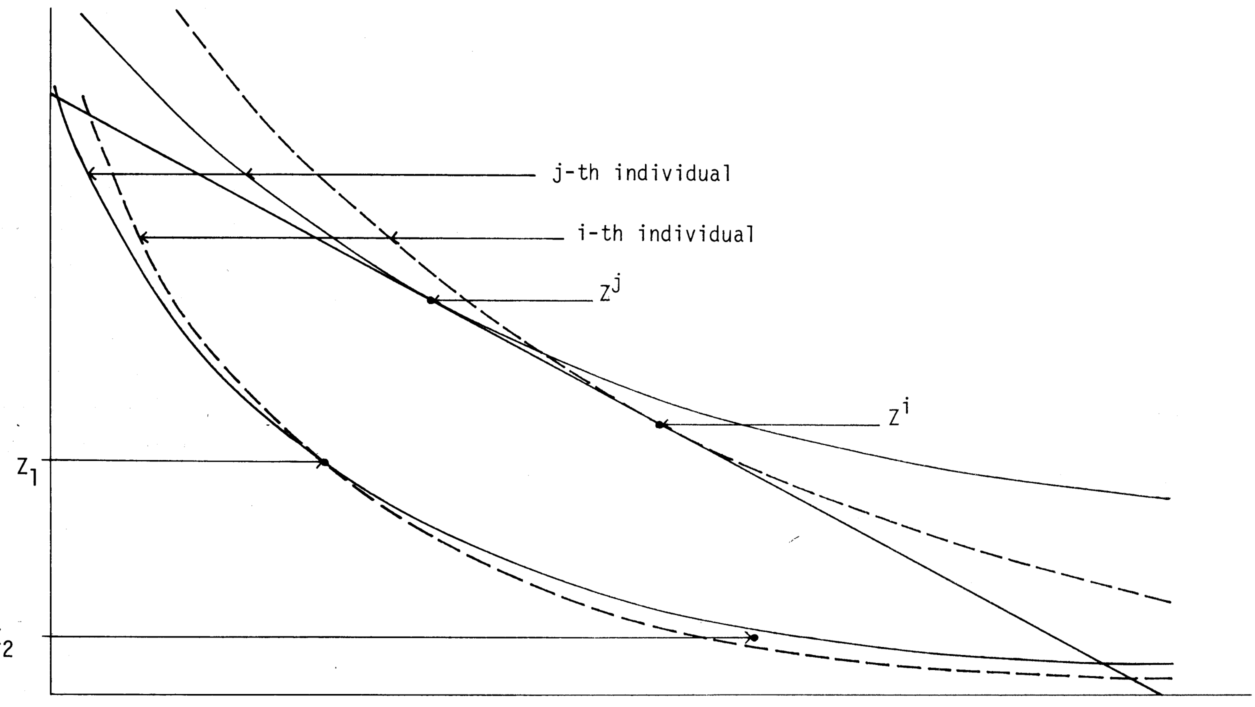
2. DISCRETE VERSUS CONTINUOUS COMMODITY SPACE

In the traditional microeconomic theory the set of alternatives or commodity space, is assumed to be a continuum. The postulated utility function for the i^{th} individual is maximized subject to the budget constraint B_i for a value z_i in the commodity space. This gives a system of demand equations

$$z_i = d(B_i, x_i, \varepsilon_i)$$

where x_i is a vector of observed characteristics of the individual and ε_i is a vector of unobserved characteristics. In applications $(B_i, x_i, \varepsilon_i)$ is observed for a sample of consumers $i = 1,2,\ldots,n$, and is utilized to estimate and test the derived demand model. The unobserved term ε_i varies from individual to individual and thus induces variations in observed demand among observationally identical consumers. By supposing that the utility function has smooth properties small variations in the utility function imply small variations in the demand. That is, the utility is maximized for a value $\bar{z} + v_i$ where v_i is a "small" term varying across the population and $\bar{z}$ is the <u>average</u> demand. Consequently, the observed demand for given B and x is distributed around a common mean $\bar{z}$ and variations in aggregate demand may therefore be interpreted as resulting from <u>common</u> variations in each of the consumers demand plus an error term. In the discrete alternative case this is no longer true since small perturbations in the utility may imply that the demand "jumps" from one alternative to another.

For instance, consider a binary choice situation with the alternatives "buy" and "not buy" and assume that the utilities for the two alternatives are close. We predict "not buy" if x is element of a set A (say) and "buy" otherwise. Now slight changes in the random terms of the utilities may give the outcomes "buy" despite $x \varepsilon A$. This may easily happen for a substantial fraction of the consumers yielding aggregate prediction which differs essentially from expected aggregate outcome.



The above utility map illustrates the case where the available alternatives are two points z_1 and z_2 in the commodity space. The indifference curves for two individuals i and j are drawn through the point x_1 and through what would have been the market points z^i and z^j if all the points under the budget line were available. We see that even if the indifference curves and the corresponding virtual market points are close, the indifference curves through z_1 might lie on different sides of the alternative z_2. Hence, on this map the i^{th} individual will choose z_1 while the j^{th} individual will choose z_2.

3. STATIC THEORIES FOR DISCRETE CHOICE

A probabilistic theory of choice behaviour is defined by i) a space of choice objects with observed attributes from a space Z and a family of subsets of such objects which are available to the decision makers, ii) observed characteristics of the decision makers belonging to a space X, iii) an individual decision rule and a distribution of decision rules across the population (cf. McFadden 1973)).

Each x-individual is assumed to have a utility function $U(z_j,x)$ which measures the preference the individual has for the j^{th} alternative with observed attribute vector z_j. In choice experiments as well as in everyday life one may notice that a decisionmaker who is presented with several observationally identical choice situations, does not necesarily make the same choice at every occasion. Many authors believe that these inconsistencies arise from the decision maker attending to different aspects of the choice situation. If one knew what caused him to attend to one aspect rather than another, a deterministic approach might be appropriate. Since these causes are unknown, we suppose the utility function is random. Moreover we assume that the utility function has the form

(3.1) $$U(z_j,x) = \sum_r \alpha_r R_r(z_j,x) + \varepsilon_j$$

where $\{R_r(z_j,x)\}$ are numerical transformations of the data called aspect components and $\{\alpha_r\}$ are weights assigned to the $R_r(z_j,x)$. $\{\varepsilon_j\}$ are the residual utilities which are not explained by the attribute vectors $\{z_j\}$. Now, since the individual in identical independent choice situations may assign the aspects different importance, we suppose that α_r and ε_j are random with expectations γ_r and zero respectively. The individual's utility function can therefore be expressed as

$$U(z_j,x) = \sum_r \gamma_r R_r(z_j,x) + \sum_r (\alpha_r-\gamma_r)R_r(z_j,x) + \varepsilon_j$$

where the first term expresses the average utility taken over repeated observationally identical independent choice experiments. So far we have only considered one particular individual. The analyst will however usually have to estimate the model from data for groups of observationally identical individuals. If the population groups are homogeneous, i.e. if all relevant information about the decision makers is contained in the x-vector the taste parameters α_r should be assumed equal for all individuals. However, this is often far from true so that in general the taste parameters should be allowed to vary across the population. In the present setting population heterogeneity can be incorporated by letting the taste parameters γ_r be random with expectations β_r. Hence, the representative individual has the utility function

(3.2) $$U(z_j,x) = v(z_j,x) + e_j(z_j,x)$$

where

$$v(z_j,x) = \sum_r \beta_r R_r(z_j,x) = \beta' R(z_j,x)$$

is the average utility and

$$e_j(z_j,x) = \sum_r (\alpha_r-\beta_r)R_r(z_j,x) + \varepsilon_j$$

is the random error term (cf. McFadden (1974), (1975), 1976a)). Without time series data and further assumptions it is therefore not possible to identify the variations due to choice inconsistencies and the variation resulting from population heterogeneity. We notice that if the variances of the α_r are large, choices between alternatives which differ substantially in their aspect components will be governed by the α-distribution while choices between similar or almost similar alternatives will be governed by the ε-distribution because the difference

$$\sum_r \alpha_r (R_r(z_j,x) - R_r(z_k,x))$$

for two almost similar alternatives j and k almost vanishes.

The selection probabilities are given by

$$(3.3)\quad P(j|x,B) = \Pr\{U(z_j,x) > \max_k \; U(z_k,x), k \neq j, k\varepsilon B\} .$$

If the distributional properties of the utility functions are specified the above expression can in principle be calculated. Thurstone (1927) assumes independent normally distributed utilities and obtains the selection probabilties in the case of two alternatives (Thurstone model). Unfortunately a generalization of this approach to the general case is cumbersome. When the number of alternatives is greater than 4 the computational difficulties in evaluating the probabilities reduces the applicability of the model. Hausman and Wise (1978) apply a three alternative Thurstone model to the analysis of transportation mode choice in Washington D.C. and they also discuss the four alternative case.

If the utilities have joint distribution of the form

$$(3.4)\quad \phi(\sum_j e^{v_j - u_j})$$

where $v_j = v(z_j,x)$ and ϕ is an arbitrary distribution function the selection probabilities turn out to be of a particularly simple form,

$$(3.5)\quad P(j|x,B) = \frac{\exp(\tau v_j)}{\sum_{k\varepsilon B} \exp(\tau v_k)}$$

where τ^2 is the common variance of the utilities. This model is known as the conditional logit model. If v_k is of the form

$$\Sigma \beta_r R_r(z_k,x) = \beta' R(z_k,x)$$

the parameter τ is not identifiable and can be suppressed by defining new parametres $\beta^* = \tau\beta$. Under weak assumptions Strauss (1979) has demonstrated that (3.4) is the only class of distribution functions implying the logit model. This is an important result and have been proved in less general versions by McFadden (1973) and Yellott (1977).

The logit model has aroused substantial interest because of its computational advantages and because it is equivalent with Luce (1959) axiom "independence from irrelevant alternatives" (IIA). In this work (1959) Luce sets out with the following problem: Let S and T be subsets of Z such that S⊂T and denote by $P_T(j)$ the probability that j is chosen from T. How do the choice probabilities $\{P_T(j), j\varepsilon T\}$ relate to the choice probabilities $\{P_S(j), j\varepsilon S\}$? To answer this question he postulated the IIA axiom which states that

$$(3.6)\quad P_T(j) = P_T(S)P_S(j), \; j\varepsilon S\subset T\subset Z$$

provided $P_Z(j) \neq 0$ and 1 for every j. This means that the relative odds in a binary choice will remain the same for these alternatives when additional alternatives become available. For instance, suppose that Z contains four alternatives. We can then pick out six subsets $T_1, \ldots, T_6$ which contain two elements each, four subsets $T_7, \ldots, T_{10}$, which contain three elements each and one set $T_{11} = Z$. For $k \leq 6$ the set $\{P_{T_k}(j); j\varepsilon T_k\}$ contains two choice probabilities, for $6 < k \leq 10$ it contains three choice probabilities and four choice probabilities for k = 11. All together this gives 28 choice probabilities. Since $\sum_{j\varepsilon T_k} P_{T_k}(j) = 1$ for each T_k the number of independent choice probabilities can be reduced to 17.

If the IIA axiom holds then the choice probabilities can be expressed by scale functions $v_j = v(z_j,x)$ which are unique up to addition by a constant. Hence choosing $v_1 = 0$ all the choice probabilities in the example above can be expressed by the three parametres v_2, v_3 and v_4.

The weakness of the axiom is that its validity depends upon whether or not the alternatives are well defined or "independent" in the sense that one alternative should not be a close substitute for another one. In the random utility function approçah this means that the utilities for different pair of alternatives should be independent or equally correlated. In many applications this is a severe restriction because there may be unobservable factors introducing asymetric dependence structure between the utilities of different alternatives. For instance in the general formulation (3.2) the error terms are dependent. Luce and Suppes (1965) and several others have criticized the axiom and given examples where it obviously does not hold. For the sake of completeness we repeat the "red-bus blue-bus" example here.

Suppose that the alternatives consist of driving car or taking a red bus. Then, a new alternative is introduced which is identical to the red bus except for the colour which is blue. If the initial frequencies of driving and riding the red bus were 2/3 and 1/3 respectively, the logit model implies that the predicted probability of driving in the three-alternative case would be 1/2. But since the blue bus alternative rather should be considered as a duplicate of the red bus alternative, we must expect that the fraction of drivers remains unchanged. It can be demonstrated that the independent Thurstone model yields approximately the same result.

Yellott (1977) has introduced an axiom which is weaker than IIA but implies IIA under the assumption that some independent random utility model is true. He calls it "invariance under uniform expansions of the choice set" (IUE). It states that the choice probabilities remain invariant when each of the choice objects are replaces by n (arbitrary) objects equivalent to the original object.

The crucial assumption here is the independence between the utilities of different alternatives. For instance, if the added objects were exact duplicates of the original objects (as it is in Yellott's example) the red-bus blue-bus example demonstrate that the independence assumption is inadequate. Moreover, in the exact duplicate case we should expect the choice probabilities to be invariant when n-1 objects identical to one of the original objects are added to the choice set.

Evidently, for the axiom to become interesting we must require that the utilities be stochastically independent accross alternatives. However, we then get the problem of defining what should be understood by equivalence between alternatives. We shall suggest the following definition: Two independent alternatives are equivalent if they have identically distributed utilities.

This means that in the long run equivalent alternatives are chosen equally often. (Recall that the expected utility is the average utility of an alternative taken over independent observationally identical choice settings.)

Yellott shows that the invariance under uniform expansions implies that

$$G(u)^n = G(u + b_n)$$

where G is the utility distribution function and b_n is a constant depending on n. He next proves that this is equivalent to

$$G(u) = \exp\{-e^{-\alpha u-\beta}\}$$

which in term, implies the choice axiom. This is the extreme value distribution and it is known from statistical theory as the asymptotic distribution of the maximum of independent identically distributed variables.

It turns out that Yellott's method can be extended to characterize the distribution of the utility processes in the dynamic case and we shall describe the essential parts of the extended theory.

4. DESCRIPTION OF A DYNAMIC MODEL FOR DISCRETE CHOICE

In the theories presented above only independent experimental choice settings are considered. However, our interest lies in analyzing the structure of discrete choices made over time. There seem to be few examples in the literature where such models have been used.

Ginsberg (1971, 1972a, b) suggests a semi-Markov approach where the transition probabilities given a transition are derived from Luce's theory. Except for the "decreasing failure rate" assumption, he does not choose the waiting time distribution from theoretical considerations. Moreover, his model requires time invariant explanatory variables. Heckmann (1978b) has recently suggested a general model for dynamic choice. His approach relies on the assumption of normally distributed utilities and permits a general correlation structure between the utilities at different times and across alternatives. (Probit model.) The probit approach becomes computationally rather cumbersome when the alternative space contains more that two alternatives. Moreover, this model is formulated in discrete time and it is not easily generalized to continuous time. The advantages are that the probit model allows very general structural and correlation patterns. Nevertheless, the choice of the normal distribution is not made from choice theoretic reasons and is therefore somewhat unsatisfactory.

Now consider an x(t)-individual at time t. This individual faces a subset $B = B_{x(t)}$ of available alternatives. This set is not necessarily the same at two different times. If the i^{th} alternative is chosen at time t, we say that the individual entered the i^{th} state at that time. Thus the state space is a subset of the alternative space. The states occupied as function of time define the individual's choice process. The choice probabilities are thought of as generated by a set of independent random utility processes $\{U_j(t)\} = \{U(z_j(t), x(t))\}$.

At this point we postulate and extended version of Yellott's axiom. IEU Axiom: "The joint probability of being in the i-th state at two or more arbitrary times is invariant under uniform expansions of the choice set".

In Dagsvik (1980) it is proved that this axiom imply that the joint distribution of the utility residuals $e_j(t_1), e_j(t_2), \ldots, e_j(t_n)$ must be of the type

$$\exp\{-G_n(x_1, x_2, \ldots, x_n)\}$$

where G_n must satisfy the invariance property

$$G_n(x_1,x_2, \ldots, x_n) = e^{-y}\, G_n(x_1-y, x_2-y, \ldots, x_n-y),$$

for all real y in addition to certain constraints. From this charactericati on it follows that the probability of choosing j at time t given that i was chosen at time s equals

$$(4.1)\quad p_{ij}(s,t) = Pr\{U_j(t) = \max_k U_k(t) \mid U_i(s)=\max_k U_k(s)\}$$

$$= \delta_{ij}(1 - \psi(t-s)) + r_j\psi(t-s)$$

where r_j is the logit function

$$r_j = \frac{e^{v_j}}{\sum_{k\varepsilon B} e^{v_k}},$$

δ_{ij} is the kronecker delta and $\psi(x)$ is an increasing function in x with $\psi(0)=0$, $\psi(\infty)=1$.

In Dagsvik (1980) it is proved that this class of choice processes contains the class of Markov processes. Recall that a process is Markovian if the lengths of stay in subsequent states are independent with exponential distribution function. Denote by Y_i the length of stay in the i-th state.

Then in the Markovian case Y_i may be expressed as

$$Y_i = \sum_{j=i}^{N_i} X_j$$

where X_j, j= 1, 2, .. are independent with common distribution function $\psi(x)$ and N_i has geometric density $(1-r_i)r_i^{n-1}$ where $\psi(x) = 1 - e^{-\theta t}$. Hence, Y_i is exponentially distributed with parameter $\theta(1-r_i)$. The intensities of the corresponding Markovian choice process are given by

$$q_{ij} = \theta r_j \text{ for } i \neq j, \; q_{ii} = -\theta(1-r_i).$$

In the above theory no state dependence effects are assumed to exist. This is obviously an unrealistic restriction in many important applications since past choice experience may influence the choice probabilities. In migration studies it is recognized that individuals tend to move back to the state originally occupied. Thus, the initial state has an additional utility component. In several applications it is reasonable to assume that the state occupied has an additional "adhesion force" utility component. This adhesion force component represents the additional attraction attributable to the efforts and costs that are connected with a transition to a new state.

In the adhesion force case mentioned above we get a Markov choice process with transition intensities

$$(4.2) \quad q_{ij} = \theta r_{ij} \text{ for } i \neq j, \; q_{ii} = -\theta(1-r_{ii})$$

where

$$(4.3) \quad r_{ij} = \frac{\exp(v_j+\mu\delta_{ij})}{\sum_{k \in B} \exp(v_k+\mu\delta_{ik})} .$$

The parameter μ represents the adhesion force utility component. This model can be made inhomogeneous by letting v_j depend on time/age.

It is important to note the difference in interpretation between the parametres θ and μ, or more generally effects due to temporal stability of the utility processes and effects due to structural state dependence (cf. Heckmann (1978a, b), Coleman (1964) and Pollak (1970)). High degree of stability of the utility processes means that previous relative utility evaluations strongly influence future choice behaviour (Habit persistence). On the other hand, in the presence of structural state dependence future choice is governed by previous choice experience. Under the Markov hypothesis and with $EU_j(t) = v_j + \mu\delta_{ij}$ we have the Markov property with respect both to habit persistence and structural state dependence effects, i.e. the transition intensities given the past choice history and the past relative utility evaluations are equal to the transition intensities given the last choice and the present relative utility evaluation.

If the state space contains only two alternatives the parameters θ and μ cannot both be identified and μ may be set equal to zero without lack of generality.

It is interesting to note that when the lengths of stay follows from individual choice according to Luce's axiom we get a Markovian choice process. This is demonstrated in the final section of Luce (1959) and we shall briefly repeat the argument here.

Let $P_{[s,t)}([\tau,t))$, $\tau\epsilon[s,t]$ denote the probability that a new choice (of state) is made in the interval $[\tau,t]$ given that a new choice is made in $[s,t]$. From the axiom it follows that

$$P_{[s,\infty)}([t,\infty)) = P_{[s,\infty)}([\tau,\infty))\ P_{[\tau,\infty)}([t,\infty)),\quad \tau\epsilon[s,t]$$

i.e., the probability of no new choice in $[s,t]$ is the product of the probability of no choice in $[s,\tau]$ and the probability of no choice in $[\tau,t]$. Differentiation with respect to t now gives

$$\lambda(s,t) \equiv \frac{\partial P_{[s,\infty)}([t,\infty))/\partial t}{P_{[s,\infty)}([t,\infty))} = \frac{\partial P_{[\tau,\infty)}([t,\infty))/\partial t}{P_{[\tau,\infty)}([t,\infty))} = \lambda(\tau,t).$$

The above expression shows that $\lambda(s,t)$ is independent of s and we obtain

$$P_{[s,\infty)}([t,\infty)) = \exp\{-\int_s^t \lambda(x)dx\}$$

which means that the choice process is Markovian.

In discrete time the corresponding transition probabilities are given by

(4.4) $\quad q_{ij} = \gamma r_{ij}$ for $i \neq j$, $q_{ii} = 1-\gamma+\gamma r_{ii}$

where $0 \leq \gamma \leq 1$. γ is a measure of the temporal stability of the utility processes.

The case $\gamma = 0$ corresponds to the case where the unobserved utility components are permanent which implies no transitions. When $\gamma = 1$ the utilities are independent at different time points and the transition matrix reduces to the logit form matrix (r_{ij}).

So far we have only considered fixed choice sets. A plausible extention to time dependent B can be defined as follows. Assume that B_t is a step function, $B_t = B_{t+1-}$, continuous from the right. This induces possible changes in the dimensions of the transition intensity matrices. To avoid this problem let $B^* = \cup_t B_t$ and let $Q^*(t) = \{q^*_{ij}(t)\}$ for $i,j \in B^*$ be the new transition defined by

$$q^*_{ij}(t) = \begin{cases} q_{ij}(t) & \text{for } i,j \in B_t \\ 0 & \text{otherwise} \end{cases}$$

5. POPULATION HETEROGENEITY. MOVER-STAYER MODELS

In several empirical studies it is noted that individuals who experience an event in the past are more likely to experience the event in the future than are individuals who have not experienced the event. The usual explanation of this empirical regularity is that utilities or/and choice possibilities are altered by past experiences. However, another explanation is that the population is heterogeneous e.g., individuals differ in some unmeasured propensity to experience the event. This propensity is to a certain extent stable over time and can induce state dependence on the aggregate level. (The spurious state dependence effect.) Without individual time series data it is not possible to distinguish between the two explanations. Ben Porath (1973) illustrates the heterogeneity problem by the following example. Suppose that a group of observationally identical individuals have an average labour force participation rate per year of 50 per cent. Two extreme interpretations are i) the group is homogeneous and each person has a probability of 50 per cent of working, or ii) the group is heterogeneous and half of the population is always working and the other half is never working. In the first case each person is expected to be working half of his working life while in the second case no transitions occur between "working" and "non-working" states. In a homogeneous population a model of individual behaviour will give the same model for the average behaviour. In the homogeneous case this is not true and it may be important to distinguish the model of individual behaviour from the aggregate model.

Several authors (see Hoem (1972) and McFarland (1970)) have argued that observed duration-dependence may be a consequence of population heterogeneity. In a heterogeneous population those who move during the first time interval differ from those who remain in that the former tend to be persons with higher propensities to move than the latter. Hence, the group of those who remain after one time period will have higher average probability of staying than the initial group and the expected fraction of the former group remaining throughout the second period is therefore larger than the expected fraction of the initial group remaining throughout the first time period. As a result the empirical evidence which follows from this type of model seems to indicate declining probabilities of movement over time. However, this is not true and the standard probability estimates are misleading because they are based upon the assumption of homogeneous subgroups. The heterogeneity in the propensity to move implies that those with high probabilities of movement move early and those who stay after some time has elapsed are low-probability-movers.

Blumen et al. (1955) modified the homogeneous Markov chain model by postulating the population to consist of two types of persons "movers" and "stayers". Their "movers" follow a discrete time Markov process while "stayers" never move. (See Singer and Spilerman (1976).) They found that this modification of the model gave a better description of the data. Later Singer and Spilerman (1972a) have generalized the "mover-stayer" model to continuous time by letting the waiting time distribution vary across the population according to a continuous probability law. In our model heterogeneity within observationally identical groups could be introduced by letting the parameters of the utility process vary. As noted in section 3 this would in general imply an extremely cumbersome estimation problem. Thus, because of mathematical difficulties it may be necessary to restrict the heterogeneity assumptions to the autocorrelation function and keep the other parameters fixed. Consequently, we may permit θ to vary across the population and assume that the probability of having a particular θ-value is given by the gamma distribution. The choice of this distribution is based on the ability of that functional form to describe a variety of unimodal curves. Let α and β be the parameters in the gamma density

$$\frac{\beta^{\alpha}\theta^{\alpha-1}e^{-\beta\theta}}{\Gamma(\alpha)}$$

In the same manner as Singer and Spilerman (1976) we could consider an analogue to the mover-stayer model letting f be the probability of being a stayer. Singer and Spilerman denote the corresponding θ-distribution the spiked gamma distribution because it adds a concentration of stayers to the gamma distribution. The above assumption lead to a non-Markovian model for the representative individual. For instance if θ has the spiked gamma distribution it can be shown that the transition probability matrix is given by

$$P(t) = fI + (1-f)(I-t\beta^{-1}R)^{-\alpha}$$

provided $R = \{r_{ij}\}$ is time constant where α and β are the parameters in the gamma density above.

6. RELAXATION OF LUCE's CHOICE AXIOM

In section three we discussed the limitation of Luce's model when some alternatives are close substitutes for each other. We described the wellknown "red-bus blue-bus" example to demonstrate that the assumption of independent utilities across alternatives obviously does not hold in this situation.

McFadden (1977a, b) and others have introduced multivariate extreme value distributions in order to permit a relaxation of Luce's choice axiom. One possible distribution has the form

$$(6.1)\quad F^{B}(x_1,x_2,\dots) = \exp\{-\sum_{i,j\in B}(e^{(v_i-x_i)/\gamma_{ij}} + e^{(v_j-x_j)/\gamma_{ij}})^{\gamma_{ij}}a_{ij}\}$$

where $a_{ij} = a_{ji} \geq 0$ and γ_{ij} measures the similarity between the alternatives i and j. These parameters are related to the correlations between the i^{th} and the j^{th} component by

$$\text{corr}\{x_i, x_j\} = 1 - \gamma_{ij}^2 \quad \text{if } a_{ik} = 0 = a_{jk},\ k \neq i,j.$$

If we assume that $(U(z_1,x), U(z_2,x), \ldots)$ has distribution function F and that $a_{ij} = 1 - \gamma_{ij}$ then the selection probabilities are given by

$$(6.2) \qquad P(j|B) = \frac{e^{v_j} + \sum_{k \neq j, k \varepsilon B} e^{v_j/\gamma_{jk}} (e^{v_j/\gamma_{jk}} + e^{v_k/\gamma_{jk}})^{\gamma_{jk}-1} (1-\gamma_{jk})}{\sum_{\substack{k,i \varepsilon B \\ k \neq i}} [e^{v_i} + (e^{v_i/\gamma_{ik}} + e^{v_k/\gamma_{ik}})^{\gamma_{ik}} (1-\gamma_{ik})]}$$

(cf. McFadden (1977a). We observe that $\gamma_{ij} = 1$ for all $i \neq j$ gives (3.5).

Now consider again the red bus-blue bus example. Let the alternatives "driving", "riding the red bus" and "riding the blue bus" be numbered 1, 2 and 3 respectively. Then $\gamma_{12} = \gamma_{13} = 1$ and $\gamma_{23} = 0$ which gives

$$P_1 = e^{v_1}/(e^{v_1} + e^{\max(v_2,v_3)}).$$

Hence we get $P_1 = 2/3$, $P_2 = P_3 = 1/6$ since $v_2 = v_3$. This is precisely the expected probability structure.

More general define G^B by

$$\log F^B(x_{i_1}, x_{i_2}, \ldots,) = G^B(x_{i_1} - v_{i_1}, x_{i_2} - v_{i_2}, \ldots) \quad i_k \varepsilon B,$$

where now F^B is a general distribution function satisfying certain conditions. Then the choice probabilities has the form

$$(6.3) \qquad P(j|B) = \partial \log G^B(v_{i_1}, v_{i_2}, \ldots)/\partial v_j \quad \text{for } j \varepsilon B,\ i_k \varepsilon B,\ k=1, 2 \ldots .$$

Now consider the dynamic case. From section 4 we notice that the transition intensities take the form

$$q_{ij} = \theta r_{ij} \qquad \text{for } i \neq j,$$

where r_{ij} are the Luce choice probabilities defined by (4.3). The natural extension of the model above to the dynamic case is to define

$$(6.4) \qquad q_{ij} = \theta \partial \log G^B(v_{r_1} + \delta_{ir_1}\mu, v_{r_2} + \delta_{ir_2}\mu, \ldots)/\partial v_j.$$

If $G^B(y_1, y_2 \ldots) = \exp\{-\sum_{j \varepsilon B} y_j\}$

we get the models of section 4.

7. ANALYSIS OF LABOUR FORCE PARTICIPATION WHEN THE CHOICE SETS ARE NOT OBSERVED

Dynamic models for the analysis of labour force participation have been applied i.a., by Ben Porath, Heckman and Willis (1977) and Heckman (1978a). Heckman and Willis assume that the participation probability for married women is constant over time but varies across observationally identical groups according to a beta distribution. The parameters of the distribution function are assumed to be loglinear functions of the vector x.

Heckman (1978a) assums a dynamic Probit model sufficiently flexible to account for population heterogeneity, true state dependence and timevarying individual characteristics.

The present approach differs from previous work. In contrast to classical labour market theory where wages are supposed to adjust until equilibrium is obtained we shall admit that due to institutional barriers and other market "imperfections" this is not necessarily so.

Now, let the alternative space consist of the two alternatives "not employed" and "employed" labeled 1 and 2, respectively. Consider a group of observationally identical not-employed persons. This group consists of an unknown fraction of persons who do not have any job opportunities and the remaining fraction who are not employed for other reasons. Thus the set of alternatives which is available for each individual is not directly observed and is therefore a source of heterogeneity. This fact motivates the introduction of random choice sets. Let B(t) denote the choice set at time t. Assuming that alternative 1 always is available B(t) can only take the values $B_1 = \{1\}$ and $B_2 = \{1, 2\}$. Notice that in contrast to common practice where transitions into and out of the labour force is observed we observe only transitions from "not employed" to "employed" and vice versa. Consequently, problems connected with the so called "discouraged worker effect" do not arise here.

A particular individual is characterized by a set of random processes $\{U_j(t), W_k(t)\}$ for j, k = 1, 2, where $U_j(t)$ measures the attractivity of the j-th alternative as perceived by the individual at time t. $W_k(t)$ is an individual productivity index determined by the market conditions in such a way that the mean of $W_2(t)$ increases with aggregate labour demand and the mean of $W_1(t)$ increases with aggregate labour supply. When $W_2(t) > W_1(t)$ the choice set B_2 is available otherwise B_1 is present. $U_j(t)$ is the utility function of the individual who chooses from B(t) while $W_k(t)$ is a "utility" function of the employers, or "market", which governs the demand for labour. Both $\{U_j(t)\}$ and $\{W_k(t)\}$ are random processes due to unobserved variables of the individual and unobserved (macro) variables characterizing the market.

The utility function $U_j(t)$ measures the "preferences" as evaluated from actual "choice" behaviour if alternative 2 is available.

Whether or not this behaviour is conditioned by other constraints is irrelevant in the present context since our prime interest is to predict the supply of labour given that B_2 is present. For the sake of simplicity we shall assume that the random components of $U_j(t)$ and $W_k(t)$ are independent. A more realistic model should allow for dependence between $U_j(t)$ and $W_k(t)$ because the aggregate supply depends upon the individual preferences. At present we assume that this dependence is accounted for by the respective deterministic components.

Let us now proceed to develop the theoretical model. When $W_2(t) > W_1(t)$ and $U_2(t) > U_1(t)$ state 2 is occupied. Otherwise state 1 is occupied. Let

(7.1) $$g_{ab}(s,t) = \Pr\{B(t) = B_b | B(s) = B_a\},$$

(7.2) $$q_{ij}(s,t) = \Pr\{U_j(t) = \max_k U_k(t) | U_i(s) = \max_k U_k(s)\},$$

(7.3) $$m_j(t) = \Pr\{W_j(t) = \max_k W_k(t)\}$$

and

(7.4) $$r_j(t) = \Pr\{U_j(t) = \max_k U_k(t)\}$$

for a, b, i, j = 1,2. Define a new state space consisting of the states 2, 3, 4 and 5.

State 2 is identical with the original state 2. State 3 is occupied if $W_1(t) > W_2(t)$ and $U_1(t) > U_2(t)$, state 4 is occupied if $W_1(t) < W_2(t)$ and $U_1(t) > U_2(t)$, state 5 is occupied if $W_1(t) > W_2(t)$ and $U_1(t) < U_2(t)$. Let $p_{ij}(s,t)$ denote the corresponding transition probabilities, i, j = 2, 3, 4 and 5. Hence

$p_{22}(s,t) = q_{22}(s,t)\ g_{22}(s,t),\ p_{23}(s,t) = g_{21}(s,t)\ q_{21}(s,t),\ p_{24}(s,\ t) = g_{22}(s,t)\ q_{21}(s,t),$

$p_{25}(s,\ t) = g_{21}(s,\ t)\ q_{22}(s,\ t),\ p_{32}(s,\ t) = g_{12}(s,\ t)\ q_{12}(8,t), p_{33}(s,\ t) = g_{11}(s,\ t) q_{11}(s,\ t),$

$p_{34}(s,\ t) = g_{12}(s,\ t)\ q_{11}(s,\ t),\ p_{35}(s,\ t) = g_{11}(s,\ t)\ q_{12}(s,\ t),\ p_{42}(s,\ t) = g_{22}(s,\ t)\ q_{12}(s,\ t),$

$p_{43}(s,\ t) = g_{21}(s,\ t)\ q_{11}(s,\ t),\ p_{44}(s,\ t) = g_{22}(s,\ t)\ q_{11}(s,\ t),\ p_{45}(s,\ t) = g_{21}(s,\ t)\ q_{12}(s,\ t),$

$p_{52}(s,\ t) = g_{12}(s,\ t)\ q_{22}(s,\ t),\ p_{53}(s,\ t) = g_{11}(s,\ t)\ q_{21}(s,\ t), p_{54}(s,\ t) =$

$g_{12}(s,\ t)\ q_{21}(s,\ t),\ p_{55}(s,\ t) = g_{11}(s,\ t)\ q_{22}(s,\ t).$

When the transition probabilities $\{q_{ij}(s,\ t)\}$ and $\{g_{ab}(s,\ t)\}$ are consistent with Markov processes, respectively, it is easily seen that the process $X(t)$ defined by $\{p_{ij}(s,\ t)\}$ is a Markov process.

Let us return to the original state space formulation where the aggregate state {3, 4, 5} is denote by state 1. We proceed to calculate the transition probabilities $h_{ij}(s,\ t)$ in the general case for $i,j=1,2$. Evidently

(7.5) $h_{22}(s,\ t) = p_{22}(s,\ t).$

The event being in state 2 at time t given that state 2 was occupied at time s has probability

$$\Pr\{\bigcup_{i\neq 2} (X(t) = 2,\ X(s) = i)\}\ /\Pr\{X(s)\neq 2\} = \sum_{i\neq 2} \frac{p_i(s)p_{i2}(s,t)}{1-m_2(s)r_2(s)}$$

where $p_i(s) = \Pr\ \{X(s) = i\}$. From above it follows that

$p_2(s) = m_2(s)\ r_2(s),\ p_3(s) = m_1(s)\ r_1(s)$

$p_4(s) = m_2(s)\ r_1(s)$ and $p_5(s) = m_1(s)\ r_2(s)$.

Consequently

$$(7.6)\qquad h_{12}(s,\ t) = \frac{r_2(t)[m_2(t)-m_2(s)g_{22}(s,t)] + m_2(s)\ g_{22}(s,t)q_{12}(s,t)}{1-m_2(s)r_2(s)}.$$

Assume next that the transition probabilities g_{ab} and q_{ij} are homogeneous i.e., they depend only on the difference t-s. Assume furthermore that $(W_k(s),\ W_k(t))$ and $(U_k(s),\ U_k(t))$ are bivariate extreme value distributed respectively. Then it follows from (4.1) that

(7.7) $g_{ij}(t) = \delta_{ij}(1-\phi(t)) + \phi(t)m_j$

and

(7.8) $q_{ij}(t) = \delta_{ij}(1-\Psi(t))+ \Psi(t)r_j$

where ϕ and Ψ are distribution function. From (6.7) and (6.8) we get

(7.9) $h_{22}(s,\ t) = (1-m_1\phi(t-s))(1-r_1\Psi(t-s))$

and

(7.10) $(1-r_2m_2)\ h_{12}(s,\ t) = r_2m_2(1-h_{22}(s,\ t)).$

We shall next consider the calculation of the likelihood function for the Markovian case when $\phi(t)=1-e^{-\eta t}$ and $\psi(t) = 1-e^{-\theta t}$. Let

$$\lambda_{ij} = p'_{ij}(0)$$

be the respective transition intensities. Differentiation yields

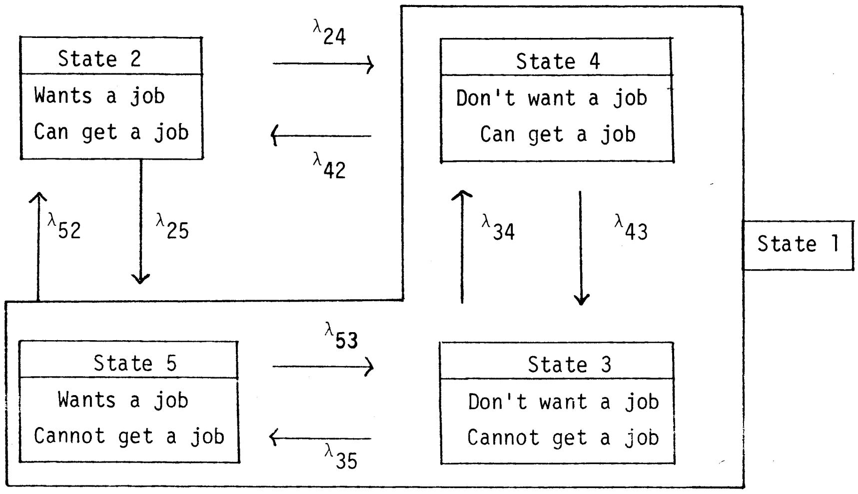
$$\lambda_{22} = -m_1\eta - r_1\theta,\ \lambda_{23} = 0,\ \lambda_{24} = r_1\theta,\ \lambda_{25} = m_1\eta,$$

$$\lambda_{32} = 0,\ \lambda_{33} = -m_2\eta - r_2\theta,\ \lambda_{34} = m_2\eta,\ \lambda_{35} = r_2\theta,$$

$$\lambda_{42} = \theta r_2,\ \lambda_{43} = m_1\eta,\ \lambda_{44} = -\theta r_2 - m_1\eta,\ \lambda_{45} = 0,$$

$$\lambda_{52} = m_2\eta,\ \lambda_{53} = r_1\theta,\ \lambda_{54} = 0,\ \lambda_{55} = -m_2\eta - r_1\theta.$$

The flow diagram of $\{X(t)\}$ is shown below



Observe that there is no direct transition between 2 and 3, and between 4 and 5. The transition intensities λ_{ij} cannot be estimated by standard methods since we only have observations on the transitions into and out of state 2.

Assume first that we have observations on $\{X(t)\}$ at discrete times t = 1, 2, ... Define

$$P_{11}(t) = \begin{pmatrix} p_{33}(t) & p_{34}(t) & p_{35}(t) \\ p_{43}(t) & p_{44}(t) & p_{45}(t) \\ p_{53}(t) & p_{54}(t) & p_{55}(t) \end{pmatrix}$$

$$P_{12}(t) = \begin{pmatrix} p_{32}(t) & 00 \\ p_{42}(t) & 00 \\ p_{52}(t) & 00 \end{pmatrix},\ P_{21}(t) = \begin{pmatrix} p_{23}(t) & p_{24}(t) & p_{25}(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ P_{22}(t) = \begin{pmatrix} p_{22}(t) & 00 \\ 0 & 00 \\ 0 & 00 \end{pmatrix}.$$

Because of the Markov property we have

$$\Pr\{X(t) = j,\ X(k) \neq 2 \text{ for } 0 \le k \le t \mid X(0) = i\}$$

$$= \sum_{j_1 \neq 2, \ldots, j_{t-1} \neq 2} p_{ij_1}(1) p_{j_1 j_2}(1) \ldots p_{j_{t-1} j}(1) = a_{ij}(t) \text{ (say)}.$$

Hence

$$a(t) = \begin{pmatrix} a_{33}(t), & a_{34}(t), & a_{35}(t) \\ a_{43}(t), & a_{44}(t), & a_{45}(t) \\ a_{53}(t), & a_{54}(t), & a_{55}(t) \end{pmatrix}$$

can be expressed as $a(t) = P_{11}(1)^t$. Introduce the vectors

$$P_2 = (p_2, 0,0),\ P_1 = (p_3, p_4, p_5),\ \underset{\sim}{1} = (1, 1, 1)'$$

and let $Y_j(t)$ be equal to one if state j ,j = 1, 2, is occupied at time t and zero otherwise. The likelihood function can now be expressed by the above formalism. We have

(7.11) $$L = \prod_i P_i^{Y_i(0)} \prod_{i,j} P_{ij}(1)^{Z_{ij}(1)} \cdot \cdot \prod_{i,j} P_{ij}(1)^{Z_{ij}(t)} \underset{\sim}{1}$$

where $Z_{ij}(t) = Y_i(t-1)\ Y_j(t)$, i, j = 1, 2.

Next, consider the problem of calculating the holding time distribution of state 1 in case we observe $\{Y_j(t)\}$ for every t. Let

$$a_{ij}(t, n) = Pr\ \{X(t) = j,\ X(\tfrac{kt}{n}) \neq 2, 0 \le k \le n \mid X(0) = i\}$$

i,j = 3, 4 and 5. We have $a(t,n) = P_{11}(\frac{t}{n})^n$. Let Λ_{ij} be the intensity matrix which corresponds to $P_{ij}(t)$. Then

$$P_{11}(\tfrac{t}{n}) = I + \tfrac{t}{n}(\Lambda_{11} + B_n)$$

where $B_n \to 0$ when $n \to \infty$. Let Γ_n be a diagonal matrix constructed from the eigenvalues of $\Lambda_{11} + B_n$. We have

$$\Lambda_{11} + B_n = C_n^{-1}\ \Gamma_n C_n$$

where C_n is the eigenvector matrix. When $n \to \infty$ $\Gamma_n \to \Gamma$, $C_n \to C$ where $\Lambda_{11} = C^{-1}\Gamma C$. Now

$$(I + \tfrac{t}{n}\Gamma_n)^n \underset{n \to \infty}{\to} \exp(t\Gamma)\ .$$

Hence

$$\lim a\ (t,n) = \lim C_n^{-1}\ (I + \tfrac{t}{n}\Gamma_n)^n C_n = C^{-1}\exp(t\Gamma)C = \exp(t\Lambda_{11}).$$

The probability density of staying in state 1 is therefore given by

(7.12) $$f_1(t) = \frac{P_1 \exp(t\Lambda_{11})}{1-m_2 r_2} \Lambda_{12}\ .$$

The probability density of staying in state 2 is given by

(7.13) $$f_2(t) = (\lambda_{25}+\lambda_{24})e^{-(\lambda_{25}+\lambda_{24})t} = (m_1 n + r_1\theta)e^{-(m_1 n + r_1 \theta)t}.$$

However, the expression for $f_1(t)$ is computationally intracable because the eigenvalue of Λ_{11} cannot be expressed in closed from. In applications it may therefore be preferable to use the likelihood function (7.11) based on the limited information sample or simply estimate the parameters by using the transition probabilities (7.9) and (7.10).

Above we have assumed that the model is identifiable. However, in case the process is observed at only two points of time it is obvious from (7.9) that without additional assumptions this is not so because h_{22} is symetrical with respect to $m_1\phi$ and $r_1\psi$. Since it is reasonable to assume that the individual preferences are more stable than the market conditions we may suppose that $\phi(t) > \Psi(t)$. In case h_{22} is known and ϕ and Ψ are exponential function the parameters are determined by algebraic equations which only have a finite number of solutions. However, we shall not proceed to discuss here the possibility of more than one acceptable solution.

So far we have not incorporated individual characteristics and other variables into the model. However, this can be done within the above framework by specifying the average utility function for the representative individual as a function of individual variables. In his analysis of female labour activity in Norway Ljones (1979) discusses the impact of various variables. Similarly, the choice set (demand) intensities c_{ij} can be specified as functions of exogeneous variables in order to account for regional variations and variations in the demand across the population. Moreover, the present model structure suggest that variables which are judged as "supply-specific" should only enter the model through the average utilities EU_j. Actual candidates may be number of and age of children. Likewise "demand-specific" variables should only enter through the intensities c_{ij}. An example of a variable of this type is the local labour market index (indicator for how well the industry mix in residental municipality is suited for female employment) introduced by Ljones (1979).

However, theories for the specification of the deterministic components of U_j and W_j are outside the scope of the present paper and will not be further discussed here. What we claim is that it is likely that a substantial part of population heterogeneity due to variations in market conditions and in individual preferences cannot be explained by the exogeneous variables. Thus, the model should be sufficiently flexible to accommodate unobserved population heterogeneity.

When time/age dependent variables are introduced we get an inhomogeneous model but if the parameters are step functions of time/age the likelihood function takes the same form as (7.11) with P_{ij} replaced by the respective time/age dependent matrices.

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