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THE UNRELIABILITY OF POPULATION FORECASTS; NUMERICAL ILLUSTRATIONS BASED ON NORWEGIAN DATA

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CONTENTS

1. INTRODUCTION AND SUMMARY

1 A. Typically, projections for closed human populations will be based on a simple recursive linear calculation procedure which can be written in the form

$$\hat{X}(t) = M(t)\, \hat{X}(t-1) \quad \text{for } t = 1,2,\ldots \tag{1}$$

Here, $M(t)$ is a square matrix of projection rates (fertility rates, survival rates, and so on), and $\hat{X}(t)$ is a vector with elements $\hat{X}_i(t)$ representing the projected number of individuals in classes into which the population has been subdivided. The recursion starts off with some known initial population vector $\hat{X}(0)$.

In a large number of cases, the purpose of making such a projection is to forecast the population $X(t)$ which will actually occur at time t. One cannot claim that the forecasting experience has been encouraging in this respect. To quote Sykes (1969), "the model has been found to give predictions of future populations which might most charitably be described as poor. Nonetheless, because of the importance of population projections to economic and social planning, they continue to be computed ...".

In this situation, it has some interest to study the properties of the deviation $X(t) - \hat{X}(t)$ under various sets of assumptions. It is the purpose of this paper to give a numerical illustration of such properties by analysing characteristic traits of its covariance matrix under two models which have appeared in the literature, and which we call the pure branching process model and the pure stochastic matrix model. We also provide a link between the two in a model for a branching process in a random environment. The numerical material previously published is scanty and mainly consists of some figures given by Sykes (1969) and Schweder (1971). We believe it to be worthwhile to take a closer look at the calculation output, and have chosen to report our results rather fully.

The outcome of our investigation is, briefly, that neither of the two models which we have studied numerically,provides a satisfactory description of real population dynamics. In both cases, the structure of the uncertainty estimated differs too badly from general forecasting experience. This conclusion will be strengthened further when our investigation is placed in a larger context in more comprehensive companion papers which are under preparation [1] , [2].

$\underset{\sim}{1}$ $\underset{\sim}{B}$. In our numerical investigations, we have concentrated on a closed female population, partitioned into 46 single-year age groups from age 0 to age 45. Our time unit (and projection unit) is one year. In line with the now classical matrix model for population dynamics, we specify the population matrix

$$\underset{\sim}{M} = \begin{pmatrix} f_0, & f_1, & f_2, \ldots, & f_{44}, & f_{45} \\ p_0, & 0, & 0, \ldots, & 0, & 0 \\ 0, & p_1, & 0, \ldots, & 0, & 0 \\ \cdot & \cdot & \cdot \; \cdots & \cdot & \cdot \\ 0, & 0, & 0, \ldots, & p_{44}, & 0 \end{pmatrix},$$

where p_x is the probability that a female who is x years old at the beginning of a given year, will survive to the end of the year, and f_x is the corresponding fertility rate. The values of these parameters may change over time in certain cases. In our data, $f_x = 0$ for $x \leqq 14$.

The projection starts at time 0, which is at the beginning of the first year, so time t is at the <u>end</u> of the t-th year, and $\hat{\underset{\sim}{X}}(t)$ is our projected population for that time point. The preceding years are counted backwards from year 1,so that year 0 is the year ending at time 0, year -1 is the next preceding year, and so on.

$\underset{\sim}{1}$ $\underset{\sim}{C}$. We have taken the end of 1967 as our "time 0", and have used the registered Norwegian female population as of December 31, 1967, as our initial population $\underset{\sim}{X}(0)$. Our fertility and mortality data have been the ones for the Norwegian female population for the years 1953 through 1968, counting only live girls born. The empirical means and standard deviations for the age-specific birth and death rates for these years may be found in Table 1. We have also computed empirical covariances between each of the 2 850 pairs of rates, but they are not displayed here.

$\underset{\sim}{1}$ $\underset{\sim}{D}$. We have tried to compare our numerical results with those published by Sykes (1969), who used a time unit of 15 years and, correspondingly, age groups of fifteen age years. Except for a rough similarity in general features, nothing much has come out of this attempt due to the large discrepancy in time and age units. We believe that our use of a unit of one year brings out the detailed features of the implications of the models much more clearly. Indeed our time unit is instrumental in revealing the essential inadequacy of the pure matrix model.

2. A PURE BRANCHING PROCESS MODEL

2 A. In the now standard matrix form of classical stable population theory, one takes the projection matrix $\underset{\sim}{M}$ as independent of t. Recursion in (1) then gives

$$(2) \qquad \hat{\underset{\sim}{X}}(t) = \underset{\sim}{M}^t \underset{\sim}{X}(0).$$

Let us take $\underset{\sim}{M}$ as given. We shall assume that the real population X(.) transfers from any time t to the subsequent time t+1 according to a multi-type branching process, as describes by Pollard (1966), Sykes (1969), and Schweder (1971). Under this model, $\underset{\sim}{X}(t)$ is a random variable with

$$E\underset{\sim}{X}(t) = \underset{\sim}{M}^t \underset{\sim}{X}(0),$$

which suggests that $\hat{\underset{\sim}{X}}(t)$ is indeed an appropriate forecast for $\underset{\sim}{X}(t)$. The covariance matrix $\underset{\sim}{C}(t)$ has elements $C_{ij}(t)$ which can be calculated from recursion relations

$$\underset{\sim}{C}(0) = \underset{\sim}{0};$$

$$(3) \qquad C_{j+1,j+1}(t+1) = p_j(1-p_j)\,\hat{X}_j(t) + p_j^2\, C_{jj}(t);$$

$$C_{j+1,k+1}(t+1) = p_j p_k\, C_{jk}(t); \quad k \neq j;$$

and similar formulas for $C_{00}(t+1)$ and $C_{0,j+1}(t+1)$; $j = 0,1,\ldots,44$ (Pollard, 1966, (18)-(25); Schweder, 1971, (2)-(6)).

2 B. In our computations based on this model, we have used the mean fertility and survival rates in Table 1 as our elements of $\underset{\sim}{M}$. Extracts from the projections can be found in Table 2. Our interest centers on the covariance matrix $\underset{\sim}{C}(t)$, however, and selected diagonal elements have been listed in panel a of Table 3. (The off-diagonal elements are much smaller.) We see quite clearly that the stochastic variation in $\underset{\sim}{X}(t)$ is mainly connected with the births in the years 1,2,...,t. There is a little additional variability due to mortality at age 0, which is substantially higher than the mortality at the other ages studied here (although still very low as compared with infant mortality in other countries). Mortality at ages 1 to 45 contributes only very little to the inherent variability of $\underset{\sim}{X}(t)$ in this model.

The numbers along each downward slope of Table 3 correspond to a birth cohort, and the trend along such a slope suggests that it might have some interest to reorganise the statistics by cohorts. This has been done in Table 4, where panel a relates to the present model. The results call for some comment:

(i) In correspondence with prior expectation, the standard deviations of the cohorts which will be born during the projection period lie on a much higher level than those of the cohorts already born at time 0.

(ii) Among the latter, the cohort of the year immediately preceding time 0 (i.e. of year 0) has higher standard deviations than earlier cohorts. This corresponds to the somewhat larger mortality at age 0.

(iii) For each cohort born prior to time 0, the standard deviations increase as the cohort grows older. This is as one would expect, because age increases as we go further into the future. The increase in variability is really even larger than the values of the standard deviations signify, because the cohorts decrease through mortality. This shows up in the selected coefficients of variation given at the bottom of panel a in Table 4. Still, the variability is really very small.

(iv) For each cohort born after time 0, there is a drop of about one unit in the standard deviation form age 0 to age 1, followed by a very small and gradual decrease. The coefficients of variation shows that this decrease is neutralized by a slightly larger decrease in the size of the cohort due to mortality, so that, for each cohort, variability, as measured by the coefficient of variation, is almost stable with respect to age.

(v) As we go along into the future, the standard deviation of the size $X_0(t)$ of the birth cohort increases somewhat. This is nice, for one would expect that it should be progressively more difficult to forecast births. The birth cohorts increase more rapidly, however, and we can observe a decrease in the coefficient of variation, in striking conflict with our notion of the reliability of birth projections. This is brought out even more clearly in Table 5.

(vi) There is another such conflict with prior notions in the material, even if one confines oneself to what one would expect internally in such a model (in contradistinction to what other sources lead us to believe). Since the size of the birth cohorts has such a lot of built-in variability, one would expect the coefficient of variation of $X_0(t)$ to get an upward shift when the cohorts born during the first years of the projection period reach the most fertile ages, and similarly for later cohorts. Nothing of this shows up in the numerical results.

(vii) Finally, there is the crucial question of the size of the $C_{ij}(t)$. Schweder (1971) has shown that we may take $X_0(t)$ as approximately normally distributed, so that

$$P\{\ |X_0(t) - \hat{X}_0(t)| \leq u_\varepsilon \sqrt{C_{00}(t)}\} \approx 1-\varepsilon, \tag{4}$$

for $0<\varepsilon<1$, where u_ε is the upper $\frac{1}{2}\varepsilon$ percentage point of the standardized normal distribution. Thus, according to our results, $X_0(t)$ would be within 1% of $\hat{X}_0(t)$ with a probability of roughly 95% for each of the first 10 projection years. Such an accuracy seems quite inconceivable in real populations. Similar conclusions follow if we study other quantities involving cohorts born during the projection period.

2 C. Just as Pollard (1968, 1970) and Sykes (1969) have done before us, and as one of us (Schweder, 1971) has said in a different context, we come to the main conclusion that the variability of $\mathbf{X}(t)$ inherent in the pure branching process model is much too small for a description of the unreliability of population forecasts. In addition to this, the model has some further undesirable consequences, the more important being that it seems to imply that the inherent inaccuracy of birth projections does not increase over time. The reason for this is, of course, that the model leaves changes in fertility rates entirely out of account.

3. THE BRANCHING PROCESS IN A RANDOM ENVIRONMENT

3 A. After reaching the above conclusion, Pollard (1968, 1970) suggested that one might use the branching process approach for each projection year, but that the branching probabilities should be regarded as random variables. This is essentially the same idea as that due to Smith and Wilkinson (1969), who study a one-dimensional Markov chain which they call a branching process in a random environment. In our case, the randomness would be caused by a randomly changing macro environment, such as weather conditions, epidemics, fads and fashions, and the like. Such influences would make $\mathbf{M}(1), \mathbf{M}(2),\ldots$ random. When these matrices are given, the conditional distribution of $\mathbf{X}(1), \mathbf{X}(2),\ldots$ is taken to be that of a time-discrete multi-type branching process, inhomogeneous in time.

In his 1968-paper, J.H. Pollard in effect limits his analysis to the case in which $\mathbf{M}(1), \mathbf{M}(2),\ldots$ are uncorrelated, identically distributed random matrices. We shall do likewise.

(In his 1970-paper, J.H. Pollard makes a first attack on the case in which the projection probabilities are random but <u>dependent</u>. We shall not go into this here. A.H. Pollard (1970) has looked more closely at random mortality fluctuations.)

3 B. Let V denote the covariance matrix operator and introduce

$$M = EM(t),$$

$$e(t) = E[X(t)|M(1),\dots, M(t)],$$

$$\bar{C}(t) = V[X(t)|M(1),\dots, M(t)],$$

$$C(t) = V[X(t)].$$

The matrix $\bar{C}(t)$ can be calculated from recurrence relations similar to (3). Evidently, by conditional expectation,

$$EX(t) = E\, e(t) = E \prod_{s=1}^{t} M(t)\, X(0) = M^{t} X(0),$$

so the projection calculated from (2) is still the expectation of $X(t)$. Furthermore,

$$C(t) = E\bar{C}(t) + V[e(t)]$$

$$= E\bar{C}(t) + V[\prod_{s=1}^{t} M(t)]X(0). \tag{4}$$

This gives a decomposition of the covariance matrix of $X(t)$ into one component generated by the branching process nature of the transition from the beginning of one projection year to the next when the branching probabilities are given, and a second component generated by the year-to-year fluctuation in the $M(s)$.

Given the likely year-to-year variation in the birth and death rates, $E\bar{C}(t)$ will probably be roughly of the size order of the covariance matrix of $X(t)$ in the pure branching process model of the previous Chapter. The second item in (4) would give the covariance matrix of $X(t)$ in a pure stochastic matrix model where

$$X(t) = M(t)\, X(t-1) \quad \text{for } t = 1,2,\dots,$$

with $M(1)$, $M(2)$,... random and independent as specified above. In this model, $\{X(t)\}$ would develop deterministically according to the linear relations above once the $M(s)$ were given.

Faced with the inadequacy of the pure branching process model as a decription of the inaccuracy of population forecasts, Sykes (1969) proceded to study the above pure stochastic matrix model. We shall do likewise.

4. THE PURE STOCHASTIC MATRIX MODEL

4 A. Under the model specified at the end of the previous Chapter, the projected population would continue to be given as in Table 1. We introduce

$$V(t) = V[\prod_{s=1}^{t} M(s)]X(0),$$

and denote the non-zero elements of $\underset{\sim}{M}(t)$ by $f_x(t)$ and $p_x(t)$. These are now taken to be random variables, and we let

$$p_x = Ep_x(t),\ f_x = Ef_x(t),$$

$$\alpha_{xy} = \text{cov}[f_x(t),\ f_y(t)],$$

$$\beta_{xy} = \text{cov}[p_x(t),\ p_y(t)],$$

and

$$\gamma_{xy} = \text{cov}[p_x(t),\ f_y(t)].$$

Then the elements of the $\underset{\sim}{V}(t)$ satisfy the recurrence relations

$$\underset{\sim}{V}(0) = 0,$$

$$V_{00}(t+1) = \sum_{i,j} \{\alpha_{ij}[V_{ij}(t) + \hat{X}_i(t)\ \hat{X}_j(t)] + f_i f_j V_{ij}(t)\},$$

$$V_{0,i+1}(t+1) = \sum_j \{\gamma_{ij}[V_{ij}(t) + \hat{X}_i(t)\ \hat{X}_j(t)] + f_j p_i V_{ij}(t)\},$$

and

$$V_{i+1,j+1}(t+1) = \beta_{ij}\{V_{ij}(t) + \hat{X}_i(t)\ \hat{X}_j(t)\} + p_i p_j V_{ij}(t),$$

for $i,j = 0,1,\ldots, 44$. These relations follow from others given by Sykes (1969).

4 B. We define the matrix $\underset{\sim}{M}$ as in Section 1 B, with the above interpretation of p_x and f_x. This matrix has 76 non-zero elements, viz. the 45 survival rates $p_0, p_1, \ldots, p_{44}$ and the 31 fertility rates $f_{15}, f_{16}, \ldots, f_{45}$. In our numerical computations, we have again used the mean rates listed in Table 1 for these elements. We have used the corresponding empirical variances and covariances for the α_{xy}, β_{xy}, and γ_{xy} above. Some selected values of $\sqrt{\alpha_{xx}}$ and $\sqrt{\beta_{xx}}$ are given in Table 1.

Evidently, this choice of $\underset{\sim}{M}$ and $V\underset{\sim}{M}(t)$ amounts to an estimation procedure, leading to a further source of projection inaccuracy. In this paper, we are not concerned with this source of variability, however, and we will take the values used for the parameters as given. (A similar remark applies to Chapter 2.)

4 C. Numerical results for this model are presented in panels b of Tables 3 and 4, and in Table 5. Comments analogous to those in (i) to (vi) of Section 2 B apply to the present case also. There is one curious feature which this model does not share with the previous one, however, viz. the dip in the standard deviation of $X_0(t)$ which now appears between times $t = 10$ and $t = 30$. We do not feel that this dip represents an interesting additional aspect of these models, and we have taken no pains to explain it.

4 D. We then turn to the study of the standard deviation of $X_0(t)$. In the pure stochastic matrix model, it is the random variation in the $\underset{\sim}{M}(s)$ which makes $\underset{\sim}{X}(t)$ a random vector. We do not really know what distribution the $\underset{\sim}{M}(s)$ have, but even if we did, we could not easily find the distribution of

$X_0(t)$ because of the complexities involved in deriving the probability distribution of $\Pi_{s=1}^{t} \underset{\sim}{M}(s)$. (Evidently, X(t) would have some generalized form of the lognormal distribution as an approximation for large t.) Thus we cannot copy (4).

Still, it seems intuitively likely that $X_0(t) - \hat{X}_0(t)$ must overshoot something like 1.5 times its standard deviation in absolute value a lot of the time. 1.5 is the upper 93.32 percentage point of the standard normal distribution, so a standard normal deviate would exceed 1.5 in absolute value with a probability of 0.1336. Although $X_0(t)$ is not normally distributed, we feel rather confident that its distribution would not be grossly different from the normal for large $\underset{\sim}{X}(0)$.

Since 1.5 times the coefficient of variation of variables like $X_0(1)$ and $X_0(2)$ is roughly 0.07, this means that corresponding relative deviations like $|X_0(1) - \hat{X}_0(1)|/\hat{X}_0(1)$ and $|X_0(2) - \hat{X}_0(2)|/\hat{X}_0(2)$ should be in excess of 7% a lot of the time. To us, this seems to represent a very high level of variability, possibly too high for the inaccuracy of one- and two-year birth projections encountered in practice. (The Norwegian projections published in 1969 overestimated the birth cohorts of its two first projection years, viz. 1969 and 1970, by as much as 3% and 10%, respectively, but it is felt that this is the result of a very real drop in fertility to a new and lower level, not a consequence of random fertility fluctuations. In any case, such shifts must surely be relatively rare in countries like ours.) We feel on shaky ground here, however, and we shall refrain from drawing firm conclusions. We would be highly interested in hearing the opinions of others on this matter.

There seems to be a straightforward explanation of this apparent overestimation of the unreliability of early birth projections. According to the arguments which Pollard (1968) and Sykes (1969) give for regarding the projection matrices as random, this randomness is supposed to catch variation in vital rates around some mean level on which they would have stayed if phenomena like cold and mild winters, droughts, epidemics and the like did not occur. There is no reason to believe that fertility variations in Norway over the period from 1953 to 1968 are of this sort only, however. There must have been an additional secular trend in the rates, i.e. the level around which there is random variation must have moved over the years. Our procedure for giving values to the α_{xy} etc. will, therefore, have picked up the variation of this mean level around the time-mean for the years, in addition to what it was supposed to reflect. A better realization of what seems to have been the intentions behind Pollard's and Sykes's suggestion, would have been to first estimate the secular trend, and then get at the (random) variation around it. They have not done this themselves (although Sykes (1969, p. 125-126) does give half a page of formulas which would have made this possible), and we have regarded it as outside our present purpose to do so.

A further discussion of possibilities for extending the Pollard-Sykes model will be given elsewhere [1], [2].

4 E. To sum up the conclusions of the present Chapter, we feel that the pure stochastic matrix model does not adequately represent the unreliability inherent in projections of births. (It seems all right concerning deaths.) The variability implied by the model for the numbers of births for the first projection years seems very high, indicating either that one can put very little trust in projected numbers of births or that the model is wrong. We regard it as a definite deficiency of the model that the coefficient of variation of $X_0(t)$ does not increase with t. It is quite contrary to what we would expect of a good model that the level of inaccuracy of birth projections implied should remain roughly constant as we progress through the projection period, as it does here.

The cause of this feature is, evidently, the constancy of the covariance matrix of the elements of $\underset{\sim}{M}(t)$, i.e. the independence of t assumed for the α_{xy}, β_{xy}, and γ_{xy}. If the increase in uncertainty about the fertility rates as one progresses into the forecasting period were to be incorporated into the model, the α_{xx} would somehow have to be made increasing functions of t. Alternatively, the introduction of correlation among the $\underset{\sim}{M}(t)$ might help.

5. FINAL REMARKS

Such changes as there are in mortality in a country like Norway cannot have much influence on the accuracy of population projections, and this seems adequately reflected by both models which we have studied.

None of them seems to give a satisfactory description of the unreliability of birth projections, however. The pure branching process implies a variability which is much too low, and the pure stochastic matrix model implies inaccuracy which either is too high, or which seems to lead one to a complete distrust in birth projections. In both cases, the model does not incorporate the increasing unreliability of birth projections as one progresses into the future.

These deficiencies are inherited, of course, by the model of Chapter 3, which combines the branching process aspect with the stochastic matrix aspect.

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Table 1. Selected input parameter values. Norwegian female population, 1953-1968

Age	Initial population (1)	Mean birth rate[x] per 10 000 (2)	Standard deviation (birth rates) per 10 000 (3)	Mean death rate per 10 000 (4)	Standard deviation (death rates) per 10 000 (5)
0	32 084			134,7	13,7
1	32 009			35,1	9,6
2	31 549			10,7	2,7
3	31 107			8,4	1,6
4	30 427			6,2	1,6
5	29 751			5,2	1,3
6	29 675			4,2	1,4
7	29 515			3,7	1,5
8	29 804			3,1	0,9
9	30 181			2,6	1,0
10	29 901			2,6	1,4
11	30 658			2,4	1,0
12	29 902			2,4	0,9
13	29 822			2,6	0,7
14	29 960			2,8	1,1
15	29 508	2,2	1,8	3,6	1,4
16	28 192	19,4	10,6	3,9	1,3
17	29 333	91,9	30,4	3,5	0,9
18	29 294	239,6	52,9	3,6	0,9
19	30 206	433,1	64,3	3,8	1,3
20	31 303	613,6	85,6	3,5	1,2
21	32 640	752,8	94,2	3,8	1,3
22	29 614	853,9	99,3	3,8	1,3
23	28 249	900,9	98,3	4,4	1,5
24	25 216	925,7	92,1	3,8	1,2
25	23 617	914,6	74,2	4,3	1,4
26	20 671	887,8	66,0	4,5	1,4
27	21 582	844,2	51,8	4,9	1,6
28	21 007	779,7	38,6	5,8	2,1
29	20 499	724,8	31,8	5,3	1,6
30	19 691	664,0	27,0	5,6	1,4
31	19 096	604,4	31,8	6,5	1,7
32	18 699	546,4	32,2	6,4	1,9
33	18 787	497,6	48,3	7,3	3,1
34	18 629	444,3	44,8	7,8	2,4
35	20 297	393,2	46,9	8,2	2,1
36	20 496	354,3	47,9	9,5	2,7
37	21 253	301,8	41,4	10,1	1,8
38	21 007	259,9	38,5	11,6	3,0
39	21 819	219,1	34,3	11,9	1,8
40	21 733	180,6	31,1	13,2	2,4
41	23 087	140,9	27,4	13,3	3,4
42	23 108	104,3	22,8	15,7	2,3
43	24 272	69,9	17,4	16,4	2,7
44	25 499	44,7	12,1	19,2	2,6
45	25 735	25,4	7,4	20,3	3,6

[x] Female babies only.

Table 2. Population in selected age groups at the end of selected projection years

Age	Initial population	Projected population										
	t=0	t=1	t=2	t=3	t=4	t=5	t=10	t=15	t=20	t=30	t=45	t=60
0	32 084	32 811	33 628	34 405	35 138	35 826	38 650	40 339	41 462	45 787	54 275	63 058
1	32 009	31 651	32 369	33 175	33 942	34 665	37 653	39 548	40 682	44 562	53 093	61 491
2	31 549	31 896	31 540	32 255	33 059	33 822	37 003	39 128	40 323	43 824	52 451	60 572
3	31 107	31 515	31 862	31 507	32 221	33 024	36 410	38 758	40 065	43 229	51 928	59 819
4	30 427	31 080	31 488	31 835	31 480	32 194	35 785	38 349	39 811	42 689	51 401	59 100
5	29 751	30 408	31 061	31 469	31 816	31 461	35 131	37 900	39 556	42 210	50 868	58 417
10	29 901	30 173	29 786	29 487	29 634	29 695	31 401	35 065	37 829	40 580	47 942	55 457
15	29 508	29 951	29 805	29 878	30 626	29 862	29 657	31 361	35 020	39 431	44 756	53 054
20	31 303	30 194	29 272	29 300	28 150	29 453	29 807	29 602	31 304	37 711	41 999	50 614
25	23 617	25 206	28 225	29 578	32 588	31 242	29 396	29 750	29 545	34 888	40 376	47 700
30	19 691	20 488	20 983	21 547	20 628	23 558	31 164	29 323	29 676	31 165	39 185	44 477
35	20 297	18 614	18 758	18 658	19 042	19 624	23 479	31 060	29 225	29 372	37 418	41 674
40	21 733	21 793	20 957	21 181	20 407	20 192	19 524	23 358	30 900	29 425	34 507	39 934
45	25 735	25 449	24 185	22 989	22 938	21 564	20 036	19 372	23 177	28 849	30 662	38 551

Table 3. Standard deviation of components of $\underset{\sim}{X}(t)$ at selected ages for selected projection years.

Age	Projection year										
	t=1	t=2	t=3	t=4	t=5	t=10	t=15	t=20	t=30	t=45	t=60
					a. Pure branching process model						
0	175,0	177,1	179,0	180,9	182,6	189,8	194,1	196,9	211,9	233,0	256,5
1	20,7	173,9	175,9	177,9	179,8	187,4	192,2	195,1	208,6	230,3	252,9
2	10,6	23,1	173,6	175,7	177,6	185,8	191,2	194,3	206,4	228,9	250,5
3	5,8	12,1	23,8	173,5	175,6	184,3	190,3	193,6	204,5	227,6	248,6
4	5,1	7,8	13,1	24,3	173,4	182,7	189,2	193,0	202,6	226,4	246,6
5	4,3	6,7	8,9	13,9	24,7	181,0	188,1	192,3	200,8	225,2	244,7
10	2,8	4,1	5,3	6,4	7,5	25,8	180,8	187,9	195,0	218,2	236,5
15	2,9	4,0	4,8	5,6	6,2	9,7	26,6	180,7	192,0	209,5	230,3
20	3,4	4,7	5,7	6,5	7,4	9,6	12,2	27,6	187,7	200,4	224,6
25	3,1	4,8	6,0	7,2	7,8	10,5	12,3	14,3	180,4	194,5	217,6
30	3,3	4,8	5,9	6,5	7,7	11,7	13,5	14,9	29,9	191,5	208,9
35	3,8	5,3	6,3	7,3	8,1	11,7	15,5	16,7	19,3	187,0	199,6
40	5,1	7,0	8,4	9,4	10,2	12,9	16,0	19,9	21,6	179,5	193,5
45	7,0	9,3	10,9	12,2	12,9	16,1	17,7	20,8	25,4	36,9	190,0
					b. Pure stochastic matrix model						
0	1534,7	1610,0	1667,8	1709,5	1734,3	1734,4	1635,5	1628,4	2016,0	2300,6	2775,3
1	44,1	1514,6	1588,9	1645,9	1687,3	1721,6	1631,6	1600,1	1945,5	2261,7	2700,8
2	30,9	53,5	1509,6	1583,7	1640,5	1725,0	1647,4	1590,5	1892,3	2246,8	2652,1
3	8,5	32,0	54,1	1508,1	1582,0	1722,2	1669,9	1587,8	1843,4	2237,9	2608,7
4	5,0	9,9	32,4	54,2	1506,9	1716,5	1686,4	1593,3	1794,5	2229,9	2565,7
5	4,8	6,9	11,0	32,6	54,5	1701,7	1702,1	1605,2	1746,9	2221,6	2523,7
10	2,9	4,1	5,8	7,1	8,1	55,0	1698,5	1698,9	1595,4	2145,8	2344,8
15	3,2	3,8	4,7	5,7	6,9	10,5	68,1	1696,5	1600,2	1972,3	2250,8
20	3,8	4,6	5,3	6,4	7,7	10,4	13,1	55,9	1693,5	1738,2	2210,6
25	3,0	5,3	6,7	8,5	9,0	11,4	13,5	15,6	1690,1	1587,4	2135,1
30	3,3	5,6	6,7	7,1	8,8	14,6	15,7	17,4	57,5	1590,3	1960,1
35	4,4	7,4	8,1	8,9	9,5	14,4	21,1	21,2	23,7	1680,7	1725,0
40	3,8	7,2	8,3	9,7	10,4	13,8	18,7	26,3	27,1	1671,8	1570,4
45	6,7	9,3	10,3	5,7	13,2	16,1	18,2	23,4	31,2	63,5	1565,0
					c. Item in panel b divided by corresponding item in panel a						
0	8,8	9,1	9,3	9,4	9,5	9,1	8,4	8,3	9,5	9,9	10,8
1	2,1	8,7	9,0	9,3	9,4	9,2	8,5	8,2	9,3	9,8	10,7
2	2,9	2,3	8,7	9,0	9,2	9,3	8,6	8,2	9,2	9,8	10,6
3	1,5	2,6	2,3	8,7	9,0	9,3	8,8	8,2	9,0	9,8	10,5
4	1,0	1,3	2,5	2,2	8,7	9,4	8,9	8,3	8,9	9,8	10,4
5	1,1	1,0	1,2	2,3	2,2	9,4	9,0	8,3	8,7	9,9	10,3
10	1,0	1,0	1,1	1,1	1,1	2,1	9,4	9,0	8,2	9,8	9,9
15	1,1	1,0	1,0	1,0	1,1	1,1	2,6	9,4	8,3	9,4	9,8
20	1,1	1,0	0,9	1,0	1,0	1,1	1,1	2,0	9,0	8,7	9,8
25	1,0	1,1	1,1	1,2	1,2	1,1	1,1	1,1	9,4	8,2	9,8
30	1,0	1,2	1,1	1,1	1,1	1,2	1,2	1,2	1,9	8,3	9,4
35	1,2	1,4	1,3	1,2	1,2	1,2	1,4	1,3	1,2	9,0	8,6
40	0,7	1,0	1,0	1,0	1,0	1,1	1,2	1,3	1,3	9,3	8,1
45	1,0	1,0	0,9	0,5	1,0	1,0	1,0	1,1	1,2	1,7	8,2

Table 4. Standard deviation of components of $\underset{\sim}{X}(t)$ at selected ages for selected birth cohorts

Projection year t	Birth year of cohort: -19	-9	-1	0[1)]	1	2	3	4	5	10	20
	a. Pure branching process model										
1	*20*[2)] 3,4	*10* 2,8	*2* 10,6	*1* 20,7	*0* 175,0						
2	*21* 4,7	*11* 4,0	*3* 12,1	*2* 23,1	*1* 173,9	*0* 177,1					
3	*22* 5,8	*12* 4,8	*4* 13,1	*2* 23,8	*2* 173,6	*1* 175,9	*0* 179,0				
4	*23* 6,7	*13* 5,5	*5* 13,9	*4* 24,3	*3* 173,5	*2* 175,7	*1* 177,9	*0* 180,9			
5	*24* 7,6	*14* 6,2	*6* 14,4	*5* 24,7	*4* 173,4	*3* 175,6	*2* 177,6	*1* 179,8	*0* 182,6		
10	*29* 11,3	*19* 9,5	*11* 16,1	*10* 25,8	*9* 173,2	*8* 175,3	*7* 177,3	*6* 179,2	*5* 181,0	*0* 189,8	
15	*34* 14,8	*24* 12,2	*16* 17,4	*15* 26,6	*14* 173,1	*13* 175,2	*12* 177,2	*11* 179,1	*10* 180,8	*5* 188,1	
20	*39* 19,0	*29* 14,7	*21* 18,9	*20* 27,6	*19* 173,0	*18* 175,1	*17* 177,1	*16* 179,0	*15* 180,7	*10* 187,3	*0* 196,9
30		*39* 21,1	*31* 22,4	*30* 29,9	*29* 172,7	*28* 174,8	*27* 176,8	*26* 178,6	*25* 180,4	*20* 187,7	*10* 195,0
45				*45* 36,9	*44* 171,5	*43* 173,6	*42* 175,7	*41* 177,7	*40* 179,5	*35* 187,0	*25* 194,5
60											*40* 193,5
	Standard deviation in per mille of projected population										
	20 1,1	*10* 0,9	*2* 3,3	*1* 6,5	*0* 53,3	*0* 52,7	*0* 52,0	*0* 51,5	*0* 51,0	*0* 49,1	*0* 48,8
	39 6,4	*39* 7,1	*31* 7,1	*45* 12,0	*44* 54,6	*43* 53,8	*42* 53,2	*41* 52,6	*40* 52,0	*35* 50,0	*40* 48,5
	b. Pure stochastic matrix model										
1	*20* 3,8	*10* 2,9	*2* 30,9	*1* 44,0	*0* 1534,7						
2	*21* 5,3	*11* 5,1	*3* 32,0	*2* 53,5	*1* 1514,6	*0* 1610,0					
3	*22* 6,5	*12* 5,8	*4* 32,4	*3* 54,1	*2* 1509,6	*1* 1588,9	*0* 1667,8				
4	*23* 7,5	*13* 6,5	*5* 32,6	*4* 54,2	*3* 1508,1	*2* 1583,7	*1* 1645,9	*0* 1709,5			
5	*24* 8,8	*14* 6,9	*6* 32,9	*5* 54,5	*4* 1506,9	*3* 1582,0	*2* 1640,5	*1* 1687,3	*0* 1734,3		
10	*29* 13,8	*19* 10,3	*11* 34,0	*10* 55,0	*9* 1503,4	*8* 1577,6	*7* 1634,9	*6* 1676,7	*5* 1701,7	*0* 1734,4	
15	*34* 19,3	*24* 13,5	*16* 34,8	*15* 55,4	*14* 1501,6	*13* 1575,6	*12* 1632,6	*11* 1673,9	*10* 1698,5	*5* 1702,1	
20	*39* 25,1	*29* 17,1	*21* 35,6	*20* 55,9	*19* 1499,0	*18* 1573,0	*17* 1630,0	*16* 1671,6	*15* 1696,5	*10* 1698,9	*0* 1628,4
30		*39* 27,1	*31* 38,5	*30* 57,5	*29* 1492,6	*28* 1566,7	*27* 1623,7	*26* 1665,2	*25* 1690,1	*20* 1680,8	*10* 1595,4
45				*45* 63,5	*44* 1470,9	*43* 1545,5	*42* 1603,4	*41* 1645,8	*40* 1671,8	*35* 1680,7	*25* 1587,4
60											*40* 1570,4
	Standard deviation in per mille of projected population										
	20 0,13	*10* 0,10	*2* 0,10	*1* 1,4	*0* 4,7	*0* 4,8	*0* 4,8	*0* 4,9	*0* 4,8	*0* 4,5	*0* 3,9
	39 0,84	*39* 0,91	*31* 1,21	*45* 2,1	*44* 4,7	*43* 4,8	*42* 4,9	*41* 4,9	*40* 4,8	*35* 4,5	*40* 3,9
	c. Item in panel b divided by corresponding item in panel a										
1	*20* 1,6	*10* 1,0	*2* 2,9	*1* 2,1	*0* 8,8						
2	*21* 1,1	*11* 1,3	*3* 2,6	*2* 2,3	*1* 8,7	*0* 9,1					
3	*22* 1,1	*12* 1,2	*4* 2,5	*3* 2,3	*2* 8,7	*1* 9,0	*0* 9,3				
4	*23* 1,1	*13* 1,2	*5* 2,3	*4* 2,2	*3* 8,7	*2* 9,0	*1* 9,3	*0* 9,4			
5	*24* 1,2	*14* 1,1	*6* 2,3	*5* 2,2	*4* 8,7	*3* 9,0	*2* 9,2	*1* 9,4	*0* 9,5		
10	*29* 1,2	*19* 1,1	*11* 2,1	*10* 2,1	*9* 8,7	*8* 9,0	*7* 9,2	*6* 9,4	*5* 9,4	*0* 9,1	
15	*34* 1,3	*24* 1,1	*16* 2,0	*15* 2,1	*14* 8,7	*13* 9,0	*12* 9,2	*11* 9,3	*10* 9,4	*5* 9,0	
20	*39* 1,3	*29* 1,2	*21* 1,9	*20* 2,0	*19* 8,7	*18* 9,0	*17* 9,2	*16* 9,3	*15* 9,4	*10* 9,1	*0* 8,3
30		*39* 1,3	*31* 1,7	*30* 1,9	*29* 8,6	*28* 9,0	*27* 9,2	*26* 9,3	*25* 9,4	*20* 9,0	*10* 8,2
45				*45* 1,7	*44* 8,6	*43* 8,9	*42* 9,1	*41* 9,3	*40* 9,3	*35* 9,0	*25* 8,2
60											*40* 8,1

1) Birth year 0 is the year immediately preceding the first projection year. Here, year 0 is 1968.

2) An italicised number states the age of the cohort at the end of the given projection year.

Table 5. Projected number of births[x] and corresponding standard deviations

Projection year t	Projected number of births[x]	Pure branching process model		Pure stochastic matrix model		Ratio of (5) to (3)
		Standard deviation	Coefficient of variation, in per mille	Standard deviation	Coefficient of variation, in per mille	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	32 811	175,0	5,3	1 535	46,8	8,8
2	33 628	177,1	5,3	1 610	47,9	9,1
3	34 405	179,0	5,2	1 668	48,5	9,3
4	35 138	180,9	5,1	1 710	48,7	9,5
5	35 826	182,6	5,1	1 734	48,4	9,5
10	38 650	189,8	4,9	1 734	44,9	9,1
15	40 339	194,1	4,8	1 636	40,6	8,4
20	41 462	196,9	4,7	1 628	39,3	8,3
30	45 787	211,9	4,6	2 016	44,0	9,5
45	54 276	233,0	4,3	2 301	42,4	9,9
60	63 059	256,5	4,1	2 775	44,0	10,8

[x] Really the number of 0-year-olds at the end of the year.